$\boxed{4b}$ **Beh** $\left(C^{0,\alpha}(\bar{\Omega}), \|\cdot\|\right)$ vollständig **Bew** Sei $(f_n)_{n\in\mathbb{N}} \subset C^{0,\alpha}(\bar{\Omega};\mathbb{R})$

CF, also $\forall \varepsilon>0\ \exists N: \forall p,q \geq N: \quad \|f_p - f_q\| = \sup_{x\in\bar{\Omega}} |f_p(x) - f_q(x)| +$

$$\sup_{x\in\bar{\Omega}} \frac{|f_p(x) - f_q(x) - f_p(y) + f_q(y)|}{|x-y|^\alpha} \overset{(*)}{<} \varepsilon. \quad \boxed{1} \text{ Aus } (*) \text{ folgt}$$

$$\sup_{x\in\bar{\Omega}} |f_p(x) - f_q(x)| < \varepsilon' \quad \forall p,q \geq N' \quad \Rightarrow \quad \forall x\in\bar{\Omega}: |f_p(x) - f_q(x)| \overset{(*)_2}{<} \varepsilon'$$

$$\forall p,q > N' \quad \Rightarrow \quad (f_p(x))_{p\in\mathbb{N}} \subset \mathbb{R} \text{ CF}, \quad \Rightarrow \quad f(x) := \lim_{p\to\infty} f_p(x)$$

$$\overset{(*)_2}{\Rightarrow} : \lim_{p\to\infty} |f_p(x) - f_q(x)| = |f(x) - f_q(x)| \Rightarrow \sup_{x\in\bar{\Omega}} |f(x) - f_q(x)| < \varepsilon'$$

$\forall x : \varepsilon >$

$$\forall q > N'$$

$$\boxed{2} \text{ Aus } (*) \text{ folgt } \sup_{x\in\bar{\Omega}} \frac{|f_p(x) - f_q(x) - f_p(y) + f_q(y)|}{|x-y|^\alpha} < \varepsilon'' \quad \forall p,q \geq N''$$

$$\Rightarrow \forall x\in\bar{\Omega}: \quad \varepsilon > \lim_{p\to\infty} \frac{|\cdots|}{|x-y|^\alpha} = \frac{|f(x) - f_q(x) - f(y) + f_q(y)|}{|x-y|^\alpha}$$

$$\Rightarrow \sup_{x \in \bar{\Omega}} \frac{|f(x) - f_q(x) - f(y) + f_q(y)|}{|x-y|^\alpha} < \varepsilon'' \qquad q > N''$$

$\boxed{3}$

$$\|f_q - f\| = \underbrace{\sup_{x \in \bar{\Omega}} |f_q(x) - f(x)|}_{\substack{< \varepsilon' \\ \boxed{1}\ \forall q > N'}} + \underbrace{\sup_{x \in \bar{\Omega}} \frac{|f_q(x) - f(x) - f_q(y) + f(y)|}{|x-y|^\alpha}}_{\substack{< \varepsilon'' \\ \boxed{2}\ \forall q > N''}}$$

$\boxed{4}$ <u>Beh</u> f stetig <u>Bew</u>: 4a) mit $Y = \mathbb{R}$ und $\boxed{1}$.

$\boxed{4c}$ Sei $f: [0,1] \to \mathbb{R}$, $f(x) = x^{1/3}$ $\qquad f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3} \frac{1}{x^{2/3}}$

<u>Beh</u>: $f \in C^{0,1/3}$. <u>Bew</u>: Zunächst $\sup_{x \in [0,1]} |f(x)| = 1$. Außerdem

$$\sup_{x,y \in [0,1]} \frac{|x^{1/3} - y^{1/3}|}{|x-y|^{1/3}} \leq 1$$ denn $||x^{1/3} - y^{1/3}|| \leq |x-y|^{1/3}$. Dazu

sei o.B.d.A. $x > y$ und man rechnet $x^{1/3} - y^{1/3} \leq (x-y)^{1/3}$

$\Leftrightarrow (x^{1/3} - y^{1/3})^3 \leq x - y \Leftrightarrow \cancel{x} - x^{2/3} y^{1/3} + x^{1/3} y^{2/3} - \cancel{y} \leq \cancel{x} - \cancel{y}$

$\Leftrightarrow x^{1/3} y^{1/3} (y^{1/3} - x^{1/3}) \leq 0$ ✓

<u>Beh</u> $f \notin C^{0,1/2}$ <u>Bew</u>: Für $x_n = 1/n$ und $y_n = 2/n$ gilt

$$\left| \frac{x_n^{1/3} - y_n^{1/3}}{(x_n - y_n)^{1/2}} \right| = \frac{(1/n)^{1/3} - (\frac{2}{n})^{1/3}}{|\frac{1}{n} - \frac{2}{n}|^{1/2}} = \frac{(1/n)^{1/3} (1 - 2^{1/3})}{(1/n)^{1/2}} \xrightarrow{n \to \infty} \infty$$