$\boxed{\text{Aufgabe 3}}$ $\boxed{A}$ <u>Beh</u> $\|u\|_{L^\infty(\mathbb{H})} \leq \|g\|_{L^\infty(\mathbb{R}^{d-1})}$ <u>Bew</u> Sei $x_d > 0$.

$$\|u(\cdot, x_d)\|_{L^\infty(\mathbb{R}^{d-1})} = \|P(\cdot, x_d) * g\|_{L^\infty(\mathbb{R}^{d-1})} \underset{\text{Young}}{\overset{\text{HÜ}}{\leq}} \|P(\cdot, x_d)\|_{L^1(\mathbb{R}^{d-1})} \cdot \|g\|_{L^\infty(\mathbb{R}^{d-1})}$$

$$\overset{\text{Blatt 6 A3}}{=} \|g\|_{L^\infty(\mathbb{R}^{d-1})}$$

$\frac{1}{p} + \frac{1}{q} = 1 + \frac{1}{r}$ $(1 \leq p, q, r \leq \infty)$

$\boxed{B}$ <u>Beh</u> Für jedes $\xi \in \mathbb{R}^{d-1}$: $\forall \varepsilon > 0\ \exists \delta' > 0$: $|(x, x_d) - (\xi, 0)| < \delta' \Rightarrow |u(x, x_d) - g(\xi)| < \varepsilon$ <u>Bew</u>: Sei $\varepsilon > 0$. $\boxed{1}$ Da g stetig ist: $\exists \delta > 0$:

$|y - \xi| < \delta \Rightarrow |g(y) - g(\xi)| < \varepsilon/2$ $\boxed{2}$ $|u(x, x_d) - g(\xi)| =$

$$\left| \int_{\mathbb{R}^{d-1}} P(x-y, x_d)\, g(y)\, dy - g(\xi) \overbrace{\int_{\mathbb{R}^{d-1}} P(x-y, x_d)\, dy}^{=1} \right| \leq \int_{\mathbb{R}^{d-1}} P(x-y, x_d)\, |g(y) - g(\xi)|\, dy$$

$$= \underbrace{\int_{B_\delta(\xi)} \cdots dy}_{\boxed{3}\ < \varepsilon/2} + \underbrace{\int_{\mathbb{R}^{d-1} \setminus B_\delta(\xi)} \cdots dy}_{\boxed{4}\ \leq \varepsilon/2} < \varepsilon$$

$\boxed{\text{Sei } \delta' := \frac{\delta}{2} \text{ und } |(x, x_d) - (\xi, 0)| < \delta'}$

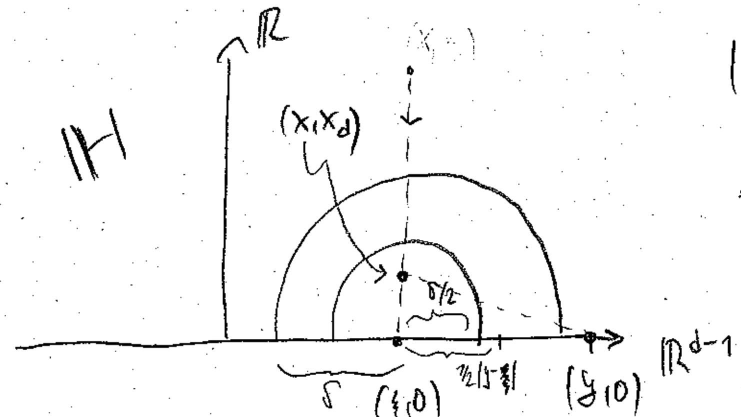
$\boxed{3}$ Wegen $\boxed{1}$ ist $\displaystyle\int_{B_\delta(\xi)} P(x-y, x_d)\,\underbrace{|g(y)-g(\xi)|}_{<\varepsilon/2}\,dy \le \frac{\varepsilon}{2}\cdot 1$

$\boxed{4}$ $\displaystyle\int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} P(x-y, x_d)\,|g(y)-g(\xi)|\,dy \underset{\text{Norm}}{\le} 2\|g\|_\infty \int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} P(x-y, x_d)\,dy$

$$= 2\|g\|_\infty \int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} \frac{2x_d}{d\omega_d\,|(x-y,x_d)|^d}\,dy = \frac{4\|g\|_\infty x_d}{d\omega_d}\int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} \frac{1}{|(x-y,x_d)|^d}\,dy$$

$$\overset{\boxed{5}}{\le} x_d\cdot\frac{4\|g\|_\infty}{d\omega_d}\int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} \frac{2^d}{|y-\xi|^d}\,dy \xrightarrow[\boxed{6}]{x_d\to 0} 0$$

$\boxed{5}$



$|y-\xi|\ge\delta \Rightarrow |(x-y, x_d)| = |(x,x_d)-(y,0)|$

$\ge \frac{1}{2}|y-\xi|$ falls $|(x,x_d)-(\xi,0)|\le\frac{\delta}{2}=\delta'$

$\Longrightarrow \dfrac{1}{|(x-y,x_d)|^d} \le \dfrac{1}{\frac{1}{2}|y-\xi|}$

$\boxed{6}$

$$\int_{\mathbb{R}^{d-1}\setminus B_\delta(\xi)} \frac{1}{|y-\xi|^d}\,dy \underset{z:=y-\xi}{=} \int_{\mathbb{R}^{d-1}\setminus B_\delta(0)} \frac{1}{|z|^d}\,dz$$

$$\leq \int_{\mathbb{R}^{d-1}\setminus B_\delta(0)} \frac{1}{|z|^d}\,dz \overset{\text{RotSym.}}{=} \int_\delta^\infty \frac{r^{d-2}}{r^d}\,dr = \int_\delta^\infty \frac{1}{r^2}\,dr = -\frac{1}{r}\Big|_\delta^\infty$$

$$= \frac{1}{\delta} > 0$$