$\boxed{\text{Aufgabe 4}}$ $(\mathbb{Z}\setminus 0,\ P,\ \mathcal{P}(\mathbb{Z}\setminus 0))$, $P(-z) = P(z)$

Bew: $\exists!\ X_0$ ZV mit $E X_0 1_C = E X 1_C \ (= 0)$ $\forall C \in \mathcal{A}_0 \Rightarrow X_0 \equiv 0$

$\mathcal{A}_0 = \sigma(h) = \sigma(\{h^{-1}(A) \mid A \in \mathcal{B}(\mathbb{R})\})$

$h(z) := |z|$, $h: \mathbb{Z}\setminus 0 \to \mathbb{R}$, $X: \mathbb{Z}\setminus 0 \longrightarrow \mathbb{R}$ $z \mapsto \operatorname{sign}(z)$

Beh $E(X 1_C) = 0 \quad \forall C \in \mathcal{A}_0$

Bew $\boxed{1}$ $z \in C \Rightarrow \exists A \in \mathcal{B}: z \in h^{-1}(A) \Rightarrow \exists a \in A: h(z) = a \Rightarrow h(-z) = a$

also $z \in C \Rightarrow \{z,-z\} \in C \Rightarrow C = \bigcup_{z \in C \cap \mathbb{N}} \{z,-z\}$

$\boxed{2}$ $E(X 1_C) = E(X 1_{\bigcup_{z \in C\cap\mathbb{N}} \{z,-z\}}) = \lim_{N\to\infty} E(X Y_N) = \lim_{N\to\infty} \int X 1_{\bigcup_{z\in C\cap\mathbb{N},\ |z|\le N}\{z,-z\}} dP$

$Y_N := 1_{\bigcup_{z \in C\cap\mathbb{N},\ z \le N} \{z,-z\}} \Rightarrow X Y_N \nearrow X 1_{\bigcup_{c \in C\cap\mathbb{N}} \{-z,z\}}$

$= \lim_{N\to\infty} \sum_{z \in C\cap\mathbb{N},\ z\le N} \int X 1_{\{z,-z\}} dP = \lim_{N\to\infty} \sum_{z \in C\cap\mathbb{N},\ z\le N} \underbrace{X(z)}_{=1} \cdot P(z) + \underbrace{X(-z)}_{=-1} P(-z) = 0$