$\boxed{\mathcal{L}^2 \not\supset \mathcal{L}^1}$ $\boxed{\mu(\Omega) < \infty}$

$\Omega = \mathbb{N}$, $\mu : \mathbb{N} \to \mathbb{R}$ $\mu(\{n\}) := \alpha_n$

$f : \mathbb{N} \to \mathbb{R}$, $\int f \, d\mu = \sum_{n=1}^{\infty} f(n)\, \mu(\{n\}) = \sum_{n=1}^{\infty} f(n)\, \alpha_n$

Sei $\alpha_n := \frac{1}{n^{p+1}}$, $f(n) := n$, $1 < p < 2$

$\int f \, d\mu = \sum n \cdot \frac{1}{n^{p+1}} = \sum \frac{1}{n^p} < \infty$ für $2 > p > 1$

$\int f^2 \, d\mu = \sum n^2 \frac{1}{n^{p+1}} = \sum \frac{1}{n^{p-1}} = \infty$ — " —

$\mu(\Omega) = \sum \frac{1}{n^{p+1}} < \infty$

$\boxed{\mathcal{L}^2 \not\subset \mathcal{L}^1}$ $\boxed{\mu(\Omega) = \infty}$ $\boxed{\underline{\text{Immer}}: \dots \subset \mathcal{L}^p \subset \mathcal{L}^{p-1} \subset \dots \subset \mathcal{L}^1, \text{ falls } \mu(\Omega) < \infty}$

$\Omega = \mathbb{N}, \quad \mu : \mathbb{N} \to \mathbb{R}, \quad \mu(\{n\}) = n^{-1/2}$

$f(n) := n^{-1/2}$

$\int f \, d\mu = \sum f(n) \, n^{-1/2} = \sum \frac{1}{n} = \infty \quad \Rightarrow \quad f \notin \mathcal{L}^1$

$\int f^2 \, d\mu = \sum (n^{-1/2})^2 \, n^{-1/2} = \sum n^{-3/2} < \infty \quad \Rightarrow \quad f \in \mathcal{L}^2$

$\mu(\Omega) = \sum \frac{1}{\sqrt{n}} = \infty$