$\boxed{2}$

$X : \Omega \to \mathbb{R} \qquad X(f) := \sum_{i=0}^{n} 1_{\{f(i)\}}(i) = \#\text{FP von } f$

$A_i := \{ f \in \Omega \mid f(i) = i \}$, $Y(f) := \sum_{i=0}^{n} 1_{A_i}(f)$, $Y : \Omega \to \mathbb{R}$

<u>Bh</u>: $X(f) = Y(f)$

<u>Bew</u>: $f \in A_i \Leftrightarrow f(i) = i$

f hat genau k FP $\Leftrightarrow f(i_1) = i_1, \dots, f(i_k) = i_k$

$\Leftrightarrow f \in A_{i_1}, f \in A_{i_2}, \dots, f \in A_{i_k} \Leftrightarrow Y(f) = k$

Außerdem, $1_{\{f(j)\}}(j) = 1 \Leftrightarrow j \in \{i_1, \dots, i_k\}$, also $X(f) = k$

(1) $E(X) = E(Y) = E\left(\sum_{i=0}^{n} 1_{A_i}\right) = \sum_{i=0}^{n} E(1_{A_i})$

$1_{A_i} : \Omega \to \mathbb{R}$, $1_{A_i}(f) = \begin{cases} 1 : f \in A_i \\ 0 : f \notin A_i \end{cases}$, $E(1_{A_i}) = 0 \cdot P(A_i^c) + 1 \cdot P(A_i) = P(A_i)$

$=$

$$= \frac{1}{|\Omega|} \cdot |\{f \in \Omega \mid f(i) = i\}| = \frac{(n-1)!}{n!}$$

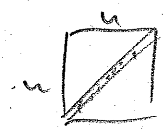
$$\Rightarrow E(Y) = \sum_{i=0}^{n} E(1_{A_i}) = n \cdot \frac{(n-1)!}{n!} = \underline{1}$$

② $$E X^2 = E(Y^2) = E\left(\left(\sum_{i=0}^{n} 1_{A_i}\right)^2\right) = E\left(\sum_{i=0}^{n} \sum_{j=0}^{n} 1_{A_i} 1_{A_j}\right)$$

$$= E\left(\sum_{\substack{i \neq j \\ 1 \leq i,j \leq n}} \underbrace{1_{A_i} 1_{A_i}} + \sum_{i=0}^{n} 1_{A_i}\right) = \sum_{\substack{i \neq j \\ 1 \leq i,j \leq n}} E(\underbrace{1_{A_{ij}}}) + \sum_{i=0}^{n} E(1_{A_i})$$

$$A_{ij} := \{f \in \Omega \mid f(i) = i,\ f(j) = j\}$$

$$= \sum_{i \neq j} \underbrace{P(A_{ij})}_{\hat{=}\ 2\,\mathrm{FP} = \frac{(n-2)!}{n!}} + 1 = (n^2 - n) \cdot \frac{(n-2)!}{n!} + 1 = n(n-1) \frac{(n-2)!}{n!} + 1$$



$$= 2$$