Aufgabe 4 ① $h: \mathbb{R} \to \mathbb{R}$, $X: \Omega \to \mathbb{R}$ ZV mit Vt

μ, also $\mu = P \circ X^{-} = X(P)$, $f: \mathbb{R} \to \mathbb{R}$ Dichte von X

1) $g = \sum \alpha_i 1_{A_i} \Rightarrow E(g(X)) = \int_\Omega g(X)\, dP = \int_\Omega g\, d\, X(P)$

$= \int_{\mathbb{R}} g\, d\mu = \int_{\mathbb{R}} \sum \alpha_i 1_{A_i}\, d\mu = \sum \alpha_i \int_{\mathbb{R}} 1_{A_i}\, d\mu = \sum \alpha_i\, \mu(A_i) \underset{\text{Dichte}}{\overset{\text{x}}{=}} \sum \alpha_i \int_{A_i} f(t)\, dt = \int_{\mathbb{R}} \sum \alpha_i 1_{A_i} f(t)\, dt$

2) $g_n \nearrow h$ monotone EltFkt $\Rightarrow E(h(X)) = \int_{\mathbb{R}} h\, d\mu$

$= \int_{\mathbb{R}} \sup g_n\, d\mu \overset{BL}{=} \sup \int_{\mathbb{R}} g_n\, d\mu = \sup \int_{\mathbb{R}} g_n f(t)\, dt$

$\overset{BL}{=} \int_{\mathbb{R}} \sup(g_n f)\, dt = \int_{\mathbb{R}} h(t) f(t)\, dt$

$\nearrow h \cdot f(t)$

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$$E(h(Y)) = \int_{\mathbb{R}} h \, d\nu = \int_{\mathbb{R}} h \, 1_S \, d\nu$$

$\boxed{S = (y_n)_{n \in \mathbb{N}}}$

S abzählbar

$$= \int h \sum_{n=1}^{\infty} 1_{\{y_n\}}(v) \, d\nu \overset{BL}{=} \sum_{n=1}^{\infty} \int h \, 1_{\{y_n\}}(v) \, d\nu$$

$$= \sum_{n=1}^{\infty} h(y_n) \, \nu\{y_n\} = \sum_{y \in S} h(y) \, \nu(\{y\})$$