$\boxed{\text{Aufgabe 1}}$ $\sigma(X_i) = \{\{X_i \in A\} \mid A \in \mathcal{B}(\mathbb{R})\}$

i) Bel X_1, X_2 unabhängig Bew: z.z.: $P(\{X_1 \in A\}) \cdot P(\{X_2 \in B\})$

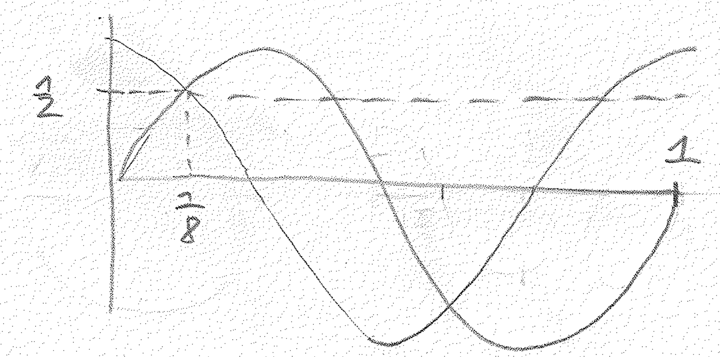
$= P(\{X_1 \in A\} \cap \{X_2 \in B\})$. Rechne:

$$P(\{X_1 \in A\} \cap \{X_2 \in B\}) = P(\{(x_1, x_2) \mid x_1 \in A, x_2 \in B\}) = P(A \times B)$$

$$= \int_{A \times B} 1 \, dP = \int_{[0,1]^2} 1_{A \times B}(x_1, x_2) \, d\lambda^2 = \int_{[0,1]^2} 1_A(x_1) \, 1_B(x_2) \, dx_1 dx_2$$

$$= \int_{[0,1]} 1_A \, dx_1 \cdot \int_{[0,1]} 1_B \, dx_2 = \lambda^1(A) \, \lambda^1(B) = P(X_1 \in A) \cdot P(X_2 \in B)$$

iii) $A := \{X_1 > \frac{1}{\sqrt{2}}\} = \{ x \in [0,1] \mid \cos 2\pi x > \frac{1}{\sqrt{2}} \} = [0, 1/8) \cup (\frac{7}{8}, 1]$



$B := \{X_2 > \frac{1}{\sqrt{2}}\} = (\frac{1}{8}, \frac{3}{8})$

$\Longrightarrow \quad P(A) = \lambda[0, 1/8) + \lambda(7/8, 1] = 1/4 = P(B)$

$P(A \cap B) = P(\emptyset) = 0$

$\Longrightarrow \quad X_1, X_2$ nicht unabhängig