$\boxed{\text{Aufgabe 2}}$ $\quad E X_k = \frac{1}{2} k^\lambda + \frac{1}{2}(-k^\lambda) = 0$, $\operatorname{Var} X_k = E X_k^2 - 0$

$= \frac{1}{2} k^{2\lambda} + \frac{1}{2} k^{2\lambda} = k^{2\lambda}$

i) $\quad s_n^2 = \sum_{k=1}^{n} \operatorname{var} X_k = \sum_{k=1}^{n} k^{2\lambda} \xrightarrow{n\to\infty} C \in \mathbb{R}$ denn $2\lambda < -1$

ii) $\quad s_n^2 = \sum_{k=1}^{n} k^{2\lambda} = \sum_{k=1}^{n} \int_{k-1}^{k} k^{2\lambda}\, dx \underset{x \le k}{\geq} \sum_{k=1}^{n} \int_{k-1}^{k} x^{2\lambda}\, dx$

$= \int_0^n x^{2\lambda}\, dx = \frac{x^{2\lambda+1}}{2\lambda+1}\Big|_0^n = \frac{n^{2\lambda+1}}{2\lambda+1}$

Beh: $\exists\, n_0 \in \mathbb{N} : \sum_{k=1}^{n} \int_{\{\frac{|X_k|}{s_n} \geq \varepsilon\}} \frac{|X_k|^2}{s_n^2}\, dP = 0 \quad \forall n \geq n_0$

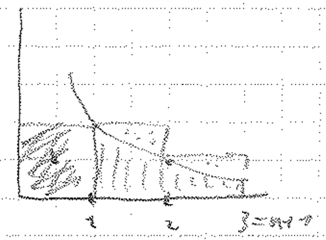
Bew: $\mathbb{1}_{\{|X_k| \geq \varepsilon s_n\}} = \begin{cases} 1 & : k^\lambda \geq \varepsilon s_n \\ 0 & : \text{sonst} \end{cases}$ $\Rightarrow$ Also: $\mathbb{1}_{\{|X_k| \geq \varepsilon s_n\}} \neq 0$

$\Rightarrow k^{2\lambda} \geq \varepsilon^2 s_n^2 \overset{k \le n}{\Longrightarrow} n^{2\lambda} \geq k^{2\lambda} \geq \varepsilon^2 s_n^2 \geq \varepsilon^2 \frac{n^{2\lambda+1}}{2\lambda+1}$

$\Rightarrow \frac{2\lambda+1}{\varepsilon^2} \geq n \quad \square$

Aus der Beh. folgt dann (L) also (ZGE).

iii) $s_n^2 = \sum_{k=1}^{n} k^{2\lambda} \underset{\text{Vgl.}}{\geq} \int_1^{n+1} x^{2\lambda}$ $\quad 2\lambda \in [-1,0]$



Wie in ii) : $k^\lambda \geq \varepsilon s_n \Rightarrow k^{2\lambda} \geq \varepsilon^2 s_n^2$

$\Rightarrow 1 \underset{\substack{\uparrow \\ 2\lambda \in [-1,0]}}{\geq} k^{2\lambda} \geq \varepsilon^2 s_n^2 \geq \varepsilon^2 \int_1^{n+1} x^{2\lambda} \underset{0 \geq 2\lambda \geq -1}{\geq} \varepsilon^2 \int_1^{n+1} \frac{1}{x}\, dx$

$= \varepsilon^2 \ln(n+1) \xrightarrow{n \to \infty} \infty$

~~$A_{k^{1/3}}$ ... $k^{-1/3}$ ...~~