Aufgabe 1

a) $\boxed{1}$ $X \geq \mu + b\sigma \iff 0 \geq \mu - X + b\sigma$

$\iff 0 \leq X - \mu - b\sigma \overset{b>0}{\iff} 0 \leq b(x-\mu) - b^2\sigma \iff b^2\sigma \leq b(x-\mu)$

$\iff b^2\sigma + \sigma \leq b(x-\mu) + \sigma \iff \sigma(1+b^2) \leq b(X-\mu) + \sigma$

$\boxed{2}$ $(bX+\sigma) - E(bX+\sigma) = bX + \sigma - bEX \overset{\text{i}}{=} bX + \sigma - b\mu$

$\overset{\boxed{1}}{\geq} b(\mu + b\sigma) + \sigma - b\mu = \cancel{b\mu} + b^2\sigma + \sigma - \cancel{b\mu} \overset{\text{ii}}{\geq} 0$

$\boxed{3}$ $P(X \geq \mu + b\sigma) \overset{\boxed{1}}{=} P(b(x-\mu) + \sigma \geq \sigma(1+b^2))$

$\overset{\boxed{2}}{=} P(\underbrace{(bX+\sigma)}_{y:=} - E(bX+\sigma) \geq \sigma(1+b^2))$

$\overset{\boxed{2}}{=} P(|y - Ey| \geq \sigma(1+b^2)) \overset{\text{Tchebby}}{\leq} \frac{1}{\sigma^2(1+b^2)^2} V(y)$

$= \frac{b^2\sigma^2}{\sigma^2(1+b^2)^2} < \frac{1}{\cancel{1}+b^2}$ $\boxed{\iff b^2 < 1 + b^2}$

b) OBdA sei $d > 0$. Sonst macht die Aussage keinen Sinn

$$P(\mu - d\sigma < X < \mu + d\sigma) = P(-d\sigma < X - \mu < d\sigma)$$

$$= P(|X-\mu| < d\sigma) = 1 - P(|X-\mu| \geq d\sigma)$$

$$\overset{TS}{\geq} 1 - \frac{1}{(d\sigma)^2} V(X) = 1 - \frac{1}{d^2}$$

c) $$P(X > x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = \int_x^\infty \frac{1}{\sqrt{2\pi}} \frac{t}{t} e^{-\frac{t^2}{2}}\, dt$$

$$\leq \frac{1}{x\sqrt{2\pi}} \int_x^\infty t\, e^{-t^2/2}\, dt = \frac{1}{x\sqrt{2\pi}} -e^{-t^2/2}\Big|_x^\infty$$

$t \geq x \Rightarrow \frac{1}{t} \leq \frac{1}{x}$

$x > 0$

$$= \frac{1}{x\sqrt{2\pi}} \left(0 - \left(-e^{-x^2/2}\right)\right) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Aufgabe 2

$$E(X^{2n}) = \int_{-\infty}^{\infty} x^{2n} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{x^{2n-1}}_{\downarrow} \underbrace{x e^{-x^2/2}}_{\uparrow}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left(\left(x^{2n-1} \left(-e^{-\frac{x^2}{2}}\right)\right)\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} (2n-1)\, x^{2n-2}\, e^{-x^2/2}\, dx \right.$$

$$= \frac{1}{\sqrt{2\pi}} (2n-1) \int_{\mathbb{R}} x^{2n-2} e^{-x^2/2}\, dx = \dots = (2n-1)(2n-3) \cdot \dots \cdot 1 \overset{!}{=} \frac{(2n)!}{2^n \cdot n!} \quad \boxtimes$$

$\boxtimes$ $n \mapsto n+1$:

$$(2(n+1)-1)(2(n+1)-3) \dots 1 = \frac{(2(n+1))!}{2^{n+1} \cdot (n+1)!}$$

$$\Leftrightarrow \quad (2n+1)(2n-1) \cdot \dots \cdot 1 = \frac{(2n+2)!}{2^n \cdot 2 \cdot n! \, (n+1)}$$

$$\overset{IV}{\Leftrightarrow} \quad 2n+1 = \frac{(2n+1)(2n+2)}{2(n+1)} \quad \checkmark$$

$$E(X^{2n+1}) = \int_{-\infty}^{\infty} x^{2n} \underbrace{x \, e^{-x^2/2}}_{} \frac{1}{\sqrt{\ }} \, dx \overset{\text{s. oben}}{=} (2n) \int_{-\infty}^{\infty} x^{2n-1} e^{-x^2/2} \, dx$$

$$= 2n \int_{-\infty}^{\infty} x^{2n-2} x \, e^{-x^2/2} \, dx = (2n)(2n-2)\cdots 2 \underbrace{\int_{-\infty}^{\infty} x \, e^{-x^2/2} \, dx}_{= E(X) = 0} = 0$$

p_Integrierbarkeit von $N(0,1)$ vt. ZV:

Für $k \in \mathbb{N}_0$ und $t > 0$ gilt: $\frac{1}{k!} t^k < e^t$ aus der Potenzreihe von e^x. Also f. $p > 0$: $e^{-t} < \frac{k!}{t^k} \overset{p>0}{\underset{x>0}{\Longleftrightarrow}}$ ~~[illegible]~~

$$|x|^p e^{-t} < \frac{k!}{t^k} |x|^p \underset{t = \frac{x^2}{2}}{\Longleftrightarrow} |x|^p e^{-\boxed{\frac{x^2}{2}}} \le \frac{k!}{\left(\frac{x^2}{2}\right)^k} |x|^p = \frac{2^k k!}{|x|^{2k}} |x|^p = 2^k k! \, |x|^{p-2k}$$

Für k groß genug gilt $|x|^p e^{-\frac{x^2}{2}} \le 2^k k! \, |x|^\alpha$ und $\alpha < -1$. Die rS. ist aber auf $\mathbb{R} \setminus [-1,1]$ λ^1-intbar.

Charakteristische Funktion

$\mu : (\mathbb{R}^n, \mathcal{B}_n) \to [0,1]$ W-Maß

$\hat{\mu} : \mathbb{R}^n \to \mathbb{C} \qquad \hat{\mu}(\underline{t}) := \int_{\mathbb{R}^n} e^{i\langle \underline{t}, x\rangle}\, d\mu \qquad t \in \mathbb{R}^n$

$$\begin{tikzcd}
(\Omega, \Sigma) \arrow[r, "X"] \arrow[d, "P"'] & (\mathbb{R}^n, \mathcal{B}_n) \arrow[dl, bend left, "PX^{-1}"] \\
{[0,1]} &
\end{tikzcd}$$

$$\varphi_X(\underline{t}) := \widehat{PX^{-1}}(\underline{t}) = \int_{\mathbb{R}^n} e^{i\langle \underline{t}, x\rangle}\, d PX^{-1} = \int_\Omega e^{i\langle \underline{t}, X\rangle}\, dP$$

Aussagen: $\to \hat{\mu}(t) = \hat{\nu}(t) \iff \mu = \nu$

$\to$ Bochner Radon: $f : \mathbb{R}^d \to \mathbb{C}$ stetig pos semidef $\Rightarrow \exists \mu : f = \hat{\mu}$

$\boxed{\to n=1: \varphi_X'(t)|_{t=0} = i\,E(X), \quad \varphi_X''(0) = -E(X^2)}$

$\Rightarrow X : \Omega \to \mathbb{R}^n,\ E(|X|^m) < \infty : \frac{\partial^m}{\partial x_{j_1} \cdots \partial x_{j_m}} \varphi_X(t) = i^m E(X_{j_1} \cdots X_{j_m} e^{i\langle t, X\rangle})$

$\boxed{\text{Aufgabe 2b}}$ i) Lösen

$$\begin{tikzcd} \mathbb{R}^n \arrow[r, "T"] \arrow[d, "P"'] & \mathbb{R}^m \arrow[dl, bend left] \\ {[0,1]} & \end{tikzcd}$$

sei P die Standard-normalvt. auf $\mathbb{R}^n$ und $T(\underline{x}) := A\underline{x} + \underline{b}$ eine affine Abbildung. Dann <u>liefert</u> PT^{-1} <u>Normalverteilung</u>. Die Klasse der NVteilungen ist invariant unter affinen Transformationen. (2.10.(4)). Ist X d-dimensional normalvt., so ist also

$$\begin{tikzcd} \mathbb{R}^n \arrow[r, "X"] \arrow[d, "P^*"'] & \mathbb{R}^n \arrow[r, "T"] \arrow[dl, bend left] & \mathbb{R} \arrow[dll, bend left, "PX^{-1}"] \\ {[0,1]} & & \end{tikzcd}$$

PX^{-1} eine Normalvt. Also ist mit

$$T(X) = \lambda_1 X_1 + \dots + \lambda_n X_n = \langle \underline{\lambda}, X \rangle$$

für $\underline{\lambda} \in \mathbb{R}^n$ eine Normalverteilung. $N_{\bar{\mu}, \bar{\sigma}}$.

$\begin{pmatrix}\mu_1\\ \vdots \\ \mu_n\end{pmatrix}, \begin{pmatrix}\lambda_1\\ \vdots \\ \lambda_n\end{pmatrix}$

Es gilt $E(\langle \underline{\lambda}, X \rangle) = E(\underline{\lambda}^T X) = \underline{\lambda}^T \cdot E(X) = \sum_{i=1}^n \lambda_i \mu_i = \langle \underline{\lambda}, \mu \rangle = \bar{\mu}$

$\Sigma(\lambda^T X) = E((\lambda^T X - E\lambda^T X)(\lambda^T X - E\lambda^T X)^T)$ $\cancel{E((\sum \lambda_i X_i - \sum \lambda_i \mu_i)^2)}$

$\bar{V}(\lambda^T X) = \operatorname{cov}(\lambda^T X, \lambda^T X)$ $E((\sum \lambda_i \ldots$ [illegible]

$= E(\lambda$ [illegible]

$$= E\left((\lambda^T(X-EX))(\lambda^T(X-EX))^T \right) = E\left(\lambda^T (X-EX)(X-EX)^T \lambda \right)$$

$$= \lambda^T \Sigma(X) \lambda = \langle \lambda, \Sigma(X)\cdot\lambda \rangle = \bar\sigma$$

ii) Betrachte

$$\begin{tikzcd} \mathbb{R}^n \arrow[r, "X"] \arrow[rr, bend right, "\lambda^T\cdot X =: y"'] & \mathbb{R}^n \arrow[r, "T"] & \mathbb{R} \end{tikzcd} \qquad T(x) := \lambda^T\cdot x$$

Also ist y nach i) $N(\underbrace{\langle\lambda,\mu\rangle}_{=\bar\mu}, \underbrace{\langle\lambda,\Sigma\lambda\rangle}_{\bar\sigma^2})$ verteilt und hat

daher die char. Funktion $\varphi_y(t) = e^{i\bar\mu t} e^{-\frac{1}{2}\bar\sigma^2 t^2}$

f. alle $t\in\mathbb{R}$. Also gilt f. jedes $\lambda \in \mathbb{R}^n$

$$\varphi_X(\lambda) = \mathbb{E}\, e^{i\langle\lambda, X\rangle} = \mathbb{E}\, e^{iy} = \mathbb{E}\, e^{i1\cdot y} = \varphi_y(\overset{\mathbb{R}}{\underset{\Downarrow}{}}1)$$

$$= e^{i\bar\mu} e^{-\frac{1}{2}\bar\sigma^2} = e^{i\langle\lambda,\mu\rangle} e^{-\frac{1}{2}\langle\lambda,\Sigma\lambda\rangle}$$

$\square$

Beh: Ist $Y: \Omega \xrightarrow{X} \mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ wobei X ZV und $T(\underline{x}) := A\underline{x} + \underline{b}$, $A \in \mathbb{R}(m \times n)$. Dann gilt: $\varphi_{\underline{b}+AX}(t) = e^{i\langle t, \underline{b}\rangle} \varphi_X(A^T t)$

Bew $\varphi_{\underline{b}+AX}(\underline{t}) = \mathbb{E}\, e^{i\langle \underline{t}, \underline{b}+AX\rangle} = \mathbb{E}\, e^{i\langle \underline{t}, \underline{b}\rangle} e^{i\langle \underline{t}, AX\rangle} = \mathbb{E}\, e^{i\langle \underline{t}, \underline{b}\rangle} e^{i\langle A^T \underline{t}, X\rangle} = e^{i\langle \underline{t}, \underline{b}\rangle} \varphi_X(A^T \underline{t})$ $\square$

Beh $X: \Omega \to \mathbb{R}$ ist $N(\mu, \sigma^2)$ verteilt $\Rightarrow \varphi_X(t) = \exp(i\mu t - \frac{1}{2}\sigma^2 t^2)$

Bew Betrachte $\Omega \xrightarrow{Y} \mathbb{R} \xrightarrow{T} \mathbb{R}$ mit $T(x) := \mu + \sigma x$ und Y $N(0,1)$-verteilt. Dann X ist $X := T \circ Y$ $N(\mu, \sigma^2)$-vt, denn:

$$P(T \circ Y \in (-\infty, t]) = P(Y \in (-\infty, T^{-1}(t)]) = \int_{-\infty}^{T^{-1}(t)} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$= \int_{-\infty}^{\frac{t-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \underset{\substack{y = \mu + \sigma x \\ dy = \sigma dx}}{=} \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy = P(X \in (-\infty, t])$$

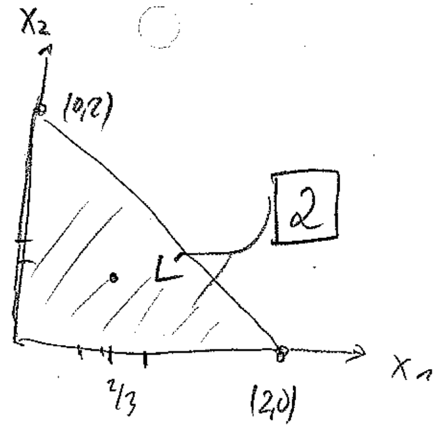
Also $\varphi_X(t) = \varphi_{T\circ y}(t) = \varphi_{\mu+\sigma Y}(t) = \varphi_Y(\sigma t) \cdot e^{i\langle t, \mu\rangle}$

$= e^{it\mu} \, e^{-i\frac{1}{2}t^2\sigma^2}$ denn $\varphi_y(t) = e^{-t^2/2}$ nach VL

$\square$

Aufgabe 3

$$\begin{tikzcd} \Omega \arrow[r, "X"] \arrow[d, "P"'] & D \arrow[r, "\pi_1"] \arrow[dl, "PX"] & \mathbb{R} \\ \mathbb{R} & & \end{tikzcd}$$



a) $$P\,1 = \int_D dPX \stackrel{!}{=} \int_D \frac{1}{N} 1_D(x_1,x_2)\, d\lambda^2 = \frac{1}{N}\int_0^2 \int_0^{2-x_1} 1 \; d\lambda(x_2)\, d\lambda(x_1)$$

$$= \frac{1}{N}\int_0^2 x_2\Big|_0^{2-x_1} d\lambda(x_1) = \frac{1}{N}\int_0^2 (2-x_1)\, d\lambda(x_1) = \frac{1}{N}\; 2x_1 - \frac{1}{2}x_1^2\Big|_0^2$$

$$= \frac{1}{N}\cdot(4-2) = \frac{2}{N} \Rightarrow N = 2 \Rightarrow f_D(x_1,x_2) = \frac{1}{2} 1_D(x_1,x_2)$$

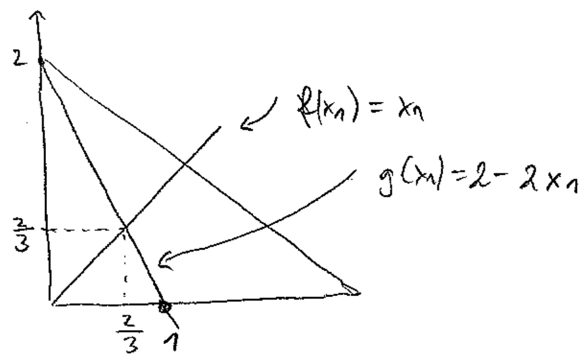
$$E X_1 = \int_\Omega \pi_1 \circ X \, dP = \int_D \pi_1 \, dPX = \int_D \pi_1(x_1,x_2)\cdot 1_D(x_1,x_2)\cdot\frac{1}{2}\, d\lambda^2$$

$$= \frac{1}{2}\int_D x_1 \, d\lambda^2 = \frac{1}{2}\int_0^2\int_0^{2-x_1} x_1 \, d\lambda(x_2)\, d\lambda(x_1) = \frac{1}{2}\int_0^2 x_1 x_2\Big|_0^{2-x_1} d\lambda(x_1)$$

$$= \frac{1}{2} \int_0^2 x_1 (2 - x_1) \, d\lambda(x_1) = \frac{1}{2} \left(x_1^2 - \frac{1}{3} x_1^3 \right) \Big|_0^2 = \frac{1}{2} \Big(\underbrace{4 - 8/3}_{= \frac{12}{3}} \Big) = \frac{4}{6} = \frac{2}{3}$$

Aus Symmetriegründen folgt $\mathbb{E}X = (\mathbb{E}X_1, \mathbb{E}X_2)^T = \left(\frac{2}{3}, \frac{2}{3}\right)$

Das ist der Schwerpunkt von D, denn



$$\Rightarrow \quad f(x_1) = g(x_1) \iff x_1 = 2 - 2x_1 \iff 2 = 3x_1$$

$$\iff \quad x_1 = \frac{2}{3}$$

$$E\left((X_1 - EX_1)(X_2 - EX_2)\right) = E X_1 X_2 - E X_1 E X_2$$

$$\operatorname{cov}(X_1, X_2) : E(X_1 X_2) = \int_\Omega (\pi_1 \circ X)\cdot(\pi_2 \circ X)\, dP = \int_\Omega (\pi_1 \cdot \pi_2)\circ X \, dP$$

$$= \int_D \pi_1 \pi_2 \, dP^X = \frac{1}{2}\int_D (\pi_1 \pi_2)(x_1, x_2)\, d\lambda^2 = \frac{1}{2}\int_D x_1 x_2 \, d\lambda^2$$

$$= \frac{1}{2}\int_0^2 \int_0^{2-x_1} x_1 x_2 \, dx_2\, dx_1 = \frac{1}{2}\int_0^2 x_1 \frac{1}{2} x_2^2 \Big|_0^{2-x_1} dx_1 = \frac{1}{4}\int_0^2 \underbrace{x_1(2-x_1)^2}\, dx_1$$

$$= x_1(4 - 4x_1 + x_1^2) = 4x_1 - 4x_1^2 + x_1^3$$

$$= \frac{1}{4}\left(\frac{4}{2}x_1^2 - \frac{4}{3}x_1^3 + \frac{1}{4}x_1^4\right)\Big|_0^2 = \frac{1}{4}\Big(\underbrace{2\cdot 4 - \frac{32}{3} + \frac{16}{4}}_{}\Big)$$

$$\frac{96}{12}$$

$$= \frac{1}{4}\left(\frac{96}{12} - \frac{128}{12} + \frac{48}{12}\right) = \frac{1}{4}\cdot\frac{16}{12} = \frac{1}{3} \Rightarrow \operatorname{cov}(X_1, X_2) = \frac{1}{3} - \left(\frac{2}{3}\right)^2 = \frac{3}{9} - \frac{4}{9} = -\frac{1}{9}$$

$$\operatorname{cov}(X_1, X_1) = \frac{1}{2}\int_0^2 \int_0^{2-x_1} x_1^2 \, dx_2\, dx_1 = \frac{1}{2}\int_0^2 \frac{1}{3}x_1^3\Big|_0^{2-x_1} dx_1 = \frac{1}{2}\int_0^2 \frac{1}{3}(2-x_1)^3 dx_1$$

$$= \frac{1}{6}\; -\frac{1}{4}(2-x_1)^4\Big|_0^2 = \frac{1}{6}\left(0 - \left(-\frac{1}{4}2^4\right)\right) = \frac{4}{6} = \frac{2}{3} \Rightarrow \Sigma(X) = \begin{pmatrix} 2/3 & -\frac{1}{9} \\ -\frac{1}{9} & 2/3 \end{pmatrix}$$

b)

$$\varphi_X(t) = E\, e^{it\cdot X} = \int_\Omega e^{it_1 X_1} e^{it_2 X_2}\, dP = \int_D e^{it_1 x_1} e^{it_2 x_2}\, d\lambda^2$$

$$= \int_0^2 \int_0^{2-x_1} \frac{1}{2} e^{it_1 x_1} e^{it_2 x_2}\, dx_2\, dx_1 = \frac{1}{it_2} \frac{1}{2} \int_0^2 e^{it_1 x_1} \left(e^{it_2(2-x_1)} - 1 \right) dx_1$$

$$\boxed{\int_0^{2-x_1} e^{it_2 x_2}\, dx_2 = \frac{1}{it_2} e^{it_2 x_2} \Big|_0^{2-x_1} = \frac{1}{it_2}\left(e^{it_2(2-x_1)} - 1\right)}$$

$$= \frac{1}{2} \frac{1}{it_2} \int_0^2 e^{ix_1(t_1-t_2)} e^{2it_2} - e^{it_1 x_1}\, dx_1$$

$$\int_0^2 e^{ix_1(t_1-t_2)}\, dx_1 = \frac{1}{i(t_1-t_2)} e^{ix_1(t_1-t_2)} \Big|_0^2 = \frac{1}{i(t_1-t_2)} \left(e^{2i(t_1-t_2)} - 1 \right)$$

$$\int_0^2 e^{it_1 x_1}\, dx_1 = \frac{1}{it_1} e^{it_1 x_1} \Big|_0^2 = \frac{1}{it_1}\left(e^{2it_1} - 1\right)$$

$$= \frac{1}{2} \frac{1}{it_2} \left[e^{2it_2} \left(\frac{1}{i(t_1-t_2)} \left(e^{2i(t_1-t_2)} - 1 \right) \right) - \frac{1}{it_1}\left(e^{2it_1} - 1\right) \right]$$

$$= \frac{1}{2} \frac{1}{it_2} \left[\frac{e^{2it_1} - e^{2it_2}}{i(t_1-t_2)} - \frac{e^{2it_1} - 1}{it_1} \right] = \frac{1}{2} \frac{1}{it_2} \left[\frac{1}{it_1(t_1-t_2)} \cdot \left(t_1 \left(e^{2it_1} - e^{2it_2} \right) - (t_1-t_2)\left(e^{2it_1} - 1\right) \right) \right]$$

$$= -\frac{1}{t_2 t_1 (t_1 - t_2)} \frac{1}{2} \left(\cancel{t_1 e^{2it_1}} - t_1 e^{2it_2} - \cancel{t_1 e^{2it_1}} + t_2 e^{2it_1} + (t_1 - t_2) \right)$$

$$= -\frac{1}{2} \frac{1}{t_1 - t_2} \left(\frac{1}{t_1} e^{2it_1} - \frac{1}{t_2} e^{2it_2} + \frac{t_1 - t_2}{t_1 t_2} \right)$$

$$= \frac{1}{2} \frac{1}{t_1 - t_2} \left(\frac{1}{t_2} e^{2it_2} - \frac{1}{t_1} e^{2it_1} - \frac{1}{t_2} + \frac{1}{t_1} \right)$$

$$\boxed{\frac{1}{t} e^{2it} = \frac{1}{t} \sum_{k \geq 0} \frac{(2it)^k}{k!}}$$

$$= \frac{1}{2} \frac{1}{t_1 - t_2} \left(\frac{1}{t_2} \sum_{k=0}^{\infty} \frac{(2it_2)^k}{k!} - \frac{1}{t_1} \sum_{k=0}^{\infty} \frac{(2it_1)^k}{k!} - \frac{1}{t_2} + \frac{1}{t_1} \right)$$

$$= \frac{1}{2} \frac{1}{t_1 - t_2} \left(\frac{1}{t_2} \sum_{k=1}^{\infty} \frac{(2it_2)^k}{k!} - \frac{1}{t_1} \sum_{k=1}^{\infty} \frac{(2it_1)^k}{k!} \right) = \frac{1}{2} \frac{1}{t_1 - t_2} \left(\sum_{k=1}^{\infty} \frac{(2i)^k t_2^{k-1}}{k!} - \sum_{k=1}^{\infty} \frac{(2i)^k t_1^{k-1}}{k!} \right) = \frac{1}{2} \frac{1}{t_1 - t_2} \left(\sum_{k=1}^{\infty} \frac{(2i)^k (t_2^{k-1} - t_1^{k-1})}{k!} \right)$$

$$= \frac{1}{2} \sum_{k=2}^{\infty} \frac{(2i)^k}{k!} \frac{t_2^{k-1} - t_1^{k-1}}{t_1 - t_2} = \frac{1}{2} \left(\underbrace{\frac{4i^2}{2!} \frac{t_2 - t_1}{t_1 - t_2}}_{= 2} + \sum_{k=3}^{\infty} \frac{t_2^{k-1} - t_1^{k-1}}{t_1 - t_2} \right)$$

$$= \frac{1}{2}\left(2 - \sum_{k=3}^{\infty} \frac{(2i)^k}{k!} \frac{t_1^{k-1} - t_2^{k-1}}{t_1 - t_2}\right)$$

$$\frac{t_1^{k-1} - t_2^{k-1}}{t_1 - t_2} = \sum_{s=0}^{k-2} t_1^{k-2-s} t_2^{s} \cdot \left(\because\right) \qquad \frac{t_1^{3-1} - t_2^{3-1}}{t_1 - t_2} = t_1 + t_2$$

$$\frac{t_1^{4-1} - t_2^{4-1}}{t_1 - t_2} = t_1^2 + t_1 t_2 + t_2^2, \qquad \frac{t_1^{5-1} - t_2^{5-1}}{t_1 - t_2} = t_1^3 + 3t_1^2 t_2 + 3t_1 t_2^2 + t_2^3$$

$$EX_1 = \frac{1}{i} \frac{\partial e_X}{\partial t_1}\Big|_{(0,0)} = -\frac{1}{2i}\left(\frac{(2i)^3}{3!} + \cancel{\ldots}\right) = -\frac{1}{2i}\left(\frac{-8i}{6} + \cancel{\ldots}\right) = \frac{2}{3}$$

$$EX_1^2 = \frac{\partial e_X}{\partial^2 t_1}\Big|_{(0,0)} = -\frac{1}{2}\frac{1}{i^2}\cdot\left(\frac{(2i)^4}{4!}\cdot 2\right) = -\frac{1}{2}\frac{1}{i^2}\frac{16}{24} = \frac{2}{3}$$

$$EX_1X_2 = \frac{\partial e_X}{\partial t_1 \partial t_2}\Big|_{(0,0)} = -\frac{1}{2}\frac{1}{i^2}\left(\frac{(2i)^4}{4!}\right) = \frac{1}{3}$$

$\square$