2. Questions on §1.3.

Question 2.1. Let K be a field, and for each pair of $r \times c$ matrices A, B over K, let

$$V[A,B] = \; K^c \overset{A}{\underset{B}{\rightrightarrows}} K^r$$

a representation of the Kronecker quiver, here denoted Q.

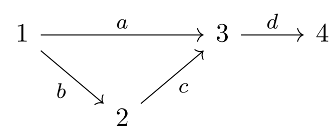
(1) Prove that any homomorphism of representations $\theta\colon V\left[\left(\begin{smallmatrix}1\\0\end{smallmatrix}\right),\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)\right] \to V\left[\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right),\left(\begin{smallmatrix}0&0\\1&0\end{smallmatrix}\right)\right]$ is uniquely determined by an element $(a,b) \in K^2$. Compute $\ker(\theta)$ and $\mathrm{cok}(\theta)$ when $(a,b) = (1,0)$ and $(a,b) = (0,1)$.

(2) Prove that any endomorphism θ of the representation $V\left[\left(\begin{smallmatrix}1&0&0\\0&1&0\end{smallmatrix}\right),\left(\begin{smallmatrix}0&1&0\\0&0&1\end{smallmatrix}\right)\right]$ is uniquely determined by an element $a \in K$. Prove that if $\theta^2 = \theta$ then either $\theta = 0$ or $\theta = 1$.

(3) Let $V(\infty) = V[(\,0\,),(\,1\,)]$ and $V(\lambda) = V[(\,1\,),(\,\lambda\,)]$ for $\lambda \in K$. For $\mu \in K \cup \{\infty\}$ prove that $V(\mu)$ is not the direct sum of two non-zero representations, and prove that $V(\mu) \cong V(\eta)$ if and only if $\mu = \eta$.

Question 2.2. Provide some missing details to the Proposition at the bottom of page 12: prove that every morphism of representations gives rise to a morphism of left KQ-modules; and check that this assignment respects composition of morphisms, addition of morphisms, and identity morphisms.

Question 2.3. Let K be a field and let Q be the quiver



(1) For each $i = 1,2,3,4$: write down a K-basis for the projective KQ-module $P[i] = KQe_i$; identify which elements of this basis lie in $e_j P[i]$; draw a diagram of K-vector spaces depicting the representation of Q corresponding to $P[i]$; and calculate the dimension vector of this representation.

(2) Write down what it means for a representation of Q to satisfy the relation $cb = a$. Give an example of a representation that satisfies this relation, and give an example that does not.