Whitehead's Theory of Extension

in Process and Reality

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1 Introduction

This paper[1] concerns Part IV of Whitehead's treatise "Process and Reality", in particular the chapters IV.II and IV.III. In Keeton (1986) one finds the following comment: *Part IV of "Process and Reality" usually causes readers of Whitehead great confusion. There is ample reason for this response. Whitehead, in fifty short pages, tries to clarify expressions of spacetime relations with which he has wrestled for more than thirty years. The effort leaves much to be desired* ... (p.315). This rating seems to be shared by many commentators. But we hope to convince the reader that most considerations in Part IV have to be seen as fundamental and profound, and not at all incomprehensible. We have to admit that some of the considerations have implications which on a first reading may look quite paradoxical (for example the idea that Democrit's atomism lies at the foundation of a Heraclitean dynamic), but this may be regarded as a special appeal. Of course, there are obvious difficulties to incorporate Part IV into a unified scheme, this cannot be denied. One reason has to be seen in Whitehead's composition of the text: whereas the reader has been promised a systematic approach, he is confronted with a rather diffuse presentation of thoughts which resembles more musical forms than scien-

[1] This is an abridged version of a text written in 2001 for the Whitehead colloquium at the University of Bielefeld. The original version (in German) is available at:

http://www.math.uni-bielefeld.de/birep/phil/pr4.pdf

The detailed discussion of some of the axioms in PR has been omitted and we have removed illustrations which were intended to provide an illumination of the considerations, since we fear that they may be misleading. One may consult the original text for a review of some standard mathematical notions, in particular basic concepts of set theory, of the foundation of set-theoretical topology (including the notion of a manifold), as well as a discussion of the relationship between algebra and geometry.

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tific explanations, with a lot of (even verbatim) repetitions, paraphrases, variations and so on.[2] Another reason has to be mentioned, and this concerns just Part IV: one has to complain about its complete conceptional isolation inside the treatise. It is isolated in two different respects: first of all, the relevant notions such a “region” are not mentioned at all in the categorical scheme, and it seems to be troublesome to assign the proper relationship to those notions which are mentioned there. Secondly, one would expect to find a discussion of fundamental notions such as “nexus” – this is indeed the case, however the decisive pages hide the notion!

Obviously, Part IV has to be considered as an independent text which has been inserted without proper adaptation. Ford (p.181) assumes that sections IV.IV.2 and IV.IV.3 have been composed already in 1926 or 1927 after discussions with de Laguna, whereas section IV.IV.4 (“strains”) may have been written as one of the last sections. Ford: *Since that doctrine depends upon a definition of straightness in terms of mere extensiveness* ..., *Whitehead felt it necessary to include in his metaphysical treatise the two mathematical chapters* (namely IV.IV.2-3) *designed to give a proper definition of straightness* (p.234). Note that Part IV follows earlier investigations of Whitehead (in particular, see PNK and CN), and a lot of formulations of these older texts are varied, but also specified and above all formalised, whereas the integration into the surrounding text remains quite unsatisfactory.

This conceptual isolation (an appreciated field of activity for many commentators – see for example the book of Ross and his discussion of the notion of perspective) has to be considered further. In particular, this has to be our main concern when we try to find the proper context for the notion of a “region”: this is one of the (few) basic notions in Part IV, but does not play any role outside of Part IV[3]. Let us add that one also misses a systematic discussion of the interplay between the notions “region”, “standpoint”, and “perspective”. On the other hand, the categorical scheme contains the notion of a “nexus”, and one would expect that it occupies a central place in Part IV[4]. But this is not the case.

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[2] And unfortunately, there are quite a lot of commentaries which follow the same principle: to repeat and concatenate fixed formulations in various permutations – formulations which are often in themselves not digestible at all: What one obtains is just Whitehead’s text send through a meet mincing machine. Even the musicality is lost in this way. Such minced meet is served for example by Hammersmith, Palter, and Ross; chopped meet by Sherburne.

[3] Whereas the “regions” considered in Part IV are related to space-time, one finds outside of Part IV formulations such as “spatial region” (PR 98, 185) or “regions in space” (PR 124) – but then one deals with abstractions which are definitely not “regions” in the sense of Part IV.

[4] See PR 461, where it is asserted that points (and more generally geometric elements) are nexuus. Note that a nexus is a set of actual entities which form a unit, for example via

In many respects, Part IV has to be seen as a foreign body inside the treatise, not only with respect to its conceptual isolation, but also looking at the abrupt change of presentation (with numbered definitions and assumptions). Of course, one may be tempted to look for a parallelism between Parts III and IV (or even III, IV and V): that Part III outlines the biological, physiological and psychological aspects, whereas Part IV deals with the mathematical and physical ones (and Part V the theological side) – but actually this would contradict the universal character of the categorical scheme, which just does not allow such a separation.

Let us start to outline the content of Part IV and its significance for Whitehead's theory. It will turn out that these considerations are quite central for the system. The word "process" in the title of the book emphasises the importance of the temporal development: time has to be considered as one of the fundamental notions of process philosophy. Also the subtitle "An essay in cosmology" should be taken seriously. The book was written on the basis of the scientific revolution created by Einstein's relativity theory on the one hand, and quantum theory on the other hand. Both theories, each one on its own, have corrected misconceptions which had been established by the scientific developments in the Early Modern Times. The Newtonian point of view was based on an assumption which was not further discussed: the possibility of working with local coordinates using real numbers, with three space axes and, independently, one time axis, as well as the global linearity. Whitehead tries to put forward a world model which encompasses both relativity theory and quantum theory. For him it is important to dwell on the question how one is able to introduce coordinates (here the method of extensive abstraction has to be named), to deal with the relationship between discrete data and continuity, and finally to discuss the meaning of simultaneity. These are the topics which are discussed in Part IV. On the basis of these considerations one should look at the possibility of interaction and influence (or, in backward direction, feelings) – but one has to be aware that the latter topics have been dealt with already in Part III, independent of the space-time model in Part IV; only the chapter IV.IV takes up the thread.

It has been stressed in the introduction of PR that Parts III and IV are the nucleus of the book: *In the third and the fourth parts, the cosmological scheme is developed in terms of its own categorical notions, and without much regard to other systems of thought* (PR vi). But one should compare this sentence which uses the key words "scheme" and "categorical" with the actual relationship between the cate-

spatial coincidence or via temporal succession – there always is a corresponding extensive quantum, thus a region – but this is mentioned there only parenthetically.

gorical scheme (as outlined in chapter I.II) and the use of it in Parts III or IV, respectively.

Extensive abstraction: The essential key word for Part IV of PR (at least for the chapters IV.II and IV.III) is that of "extensive abstraction". One may argue that it is quite hidden and occurs only as title of IV.II.III., but this is misleading. If one takes into account all the explicit references to earlier publications (PR 440, 453, 454, 455), and the relevance of the extensive abstraction in PNK as well as CN, one cannot overestimate its importance for Whitehead's cosmology. In this point, it seems that all the commentaries do agree: a large amount of pages, even complete books (see the list of references) are devoted to this topic.

Let us add a short comment on the use of the words "abstract" and "abstraction" in philosophy: it was Boethius who started to use this concept for mathematical objects derived from physical entities.

2 Regions and Connectivity (IV.II.I and IV.II.II)

We are going to sketch the essential arguments of the sections IV.II.I and IV.II.II, but we will refrain from discussing assertions of more peripheric character (in particular all those formulations which bound the validity of some structure to the "present cosmic epoch" – whatever this means).

2.1 The Title: Extensive Connection

It seems to be obvious that Whitehead wants to establish the notion of "extensive connection" as a basis for the further considerations. Connectivity refers to a topological concept – unconnectedness would mean that there is a decomposition in several components. Modern topology distinguishes between a lot of different connectivity properties (connected, locally connected, pathwise connected, and so on), it would be worthwhile to incorporate this into a Whiteheadian system. According to Whitehead, the problems concerning the extensive connection are part of his discussion of the order of nature (see PR 148). "Order" may be interpreted here both as a general concept of colloquial as well as philosophical language, but also in the mathematical sense of dealing with ordered sets, with a hierarchic relation.

Extension: Here, we have to refer to the book "Ausdehnungslehre" (theory of extension) by Hermann Grassmann, published in 1844. A second edition appeared in 1861, but both editions did not find the attention which they would have deserved. This book develops a general theory of vector spaces (vector algebra, vector analysis, tensor calculus, n-dimensional geometry). Already in his introduction to UA (1898), Whitehead referred to Grassmann as follows: *It is the purpose of this work to present a thorough investigation of the various systems of Symbolic Reasoning* *allied to ordinary Algebra. The chief examples of such systems are Hamilton's Quaternions, Grassmann's Calculus of Extensions, and Boole's Symbolic Logic*, and the structure of UA shows very clearly Whitehead’s emphasis of interpreting algebra and geometry as being concerned with “extension”, and this involves the whole context of the vector calculus - not only the (somewhat boring) theory of linear vector spaces, but the theory of vector fields[5] and thus of (partial) differential equations and of dynamical systems.

The Axiomatic Approach: Whitehead uses a kind of axiomatic presentation, with axioms and assumptions. But one should be surprised to see that he does not care to separate assertions which are plain assumptions and consequences which can be derived: Indeed, he formulates at the end: *A sufficient number of assumptions, some provable, and some axiomatic have now been stated* (PR 459). This is in sharp contrast to the classical procedure, say the setting of Euclidean geometry. There a system of axioms is required to have the following three properties: consistency, completeness and minimality[6]. The main property is clearly the consistency, since any inconsistent theory would be of no use. It is the aim of Whitehead’s theory of extension to exhibit a system of axioms which has as model the physical world (but this he formulates only at the end) and we suppose that he definitely would like to present a complete system of axioms. The first paragraphs of section I provide some hints about the role the axioms play. Whitehead insists that they could be used in different settings. Also, he insists that he does not care about minimality.

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[5] Hampe claims in “Wahrnehmung der Organismen“, that Whitehead’s notion of a vector differs from the “exact mathematical notion”, but he seems to have in mind only the naive vectors of school mathematics! Our interpretation of section IV.III.V will outline in more detail the necessary vision of vector fields. This will turn out to be an essential ingredient in order to understand Whitehead’s use of vectors.

[6] Consistency means that one cannot derive a self-contradiction, or, phrasing it differently, that there exists a model which satisfies all the axioms. Completeness asserts that there can be at most one model. Minimality means that no axiom is a consequence of the remaining ones and therefore could be deleted. In modern mathematics, the use of a system of axioms has a quite different character: First of all, one usually drops the requirement of completeness, thus allowing the possibility of a wealth of models (see for example the axiomatization of group theory, the models are the groups, all satisfy the axioms, but there are many non-isomorphic ones). Also, the requirement of minimality is not considered as being really important: it may be helpful to use a small number of axioms, but if in doubt one would prefer to work with a meaningful system of axioms, even if it is redundant.

2.2 Sections IV.II.I and IV.II.II.

The basic notions here are “region”[7] and “connectedness”, and the various possible relations between regions are illustrated by a sort of Venn diagrams. Of course, one should be aware (and Whitehead stresses this repeatedly) that this may be misleading. In particular, such diagrams usually concern point sets, whereas in the setting of the book, points will be introduced much later, namely as abstractive elements. Also, the pictures used visualise 2-dimensional sets, whereas already any space-time description needs four dimensions. In addition the various directions appear to be indistinguishable in contrast to the obvious differences between space and time axes.

As we have mentioned, there are just two basic concepts, that of a region and the connectedness relation. It should be stressed that one of the axioms (the assumption 2) asserts that any two regions A and B are mediately connected: there is always a region C such that both A and B are connected with C.

Not only the pictorial illustrations, but also the language used has a set-theoretical flavour: the relations discussed are called inclusion and overlapping, there are dissections of regions, and so on. We will come back to this setting later (under the heading Mereology), when we review the process of introducing points as abstractive elements. It seems to us that the concept of “tangentially” inclusion (definition 8) requires special care, since the use of the word “tangent” may suggest that some kind of linearization is already available – but this does not seem to be the case. The difference between tangentially and non-tangentially inclusions concerns the behaviour at the boundary (whatever this means).

Consistency: It has been noted by some commentaries that the system of axioms as presented by Whitehead has some incompatibilities: for example, assumption 4 asserts that no region is even mediately connected with itself, but this contradicts the usual interpretation of definition 1. Thus, the system of axioms, if taken serious, has to be revised – but apparently, no-one cares.

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[7] Palter (p.107) writes: *In terms of standard mathematical conceptions, regions seem to be purely topological in character.* But he adds*: Whitehead never says explicitly that his regions are closed, but it is a reasonable inference from the properties he does attribute to regions ....* This is in sharp contrast to definition 21, which asserts that all points which belong to a region (“situated” in a region) are inner points, so that regions are open, and thus in general not closed!

2.3 What are regions?

Looking at Chapters IV.II and IV.III of PR one may be surprised to find the notion of a "region" used abruptly without any further explanation[8], as a notion inside a system of axioms which gets its meaning just by these axioms*: The term "region" will be used for the relata which are involved in the scheme of "extensive connection"* (PR 449). Such an approach may be appreciated in mathematics, but even in mathematics one would like to know from the start possible applications. Let us recall what Whitehead has in mind: *the application of this theory of extension to the existing physical world* (PR 459), but one has to wait quite a while to find corresponding hints. It is section IV.III.IV which starts to discuss the physical relevance. There, in paragraph 4, we find the decisive key word: Any actual entity[9] yields a region, namely its "standpoint". The reluctance to provide hints for the interpretation is new in PR (and may be unintentional), the previous presentations are much more readable by dealing directly with "events"[10]. Actually, at the beginning of Part IV one finds more detailed information: IV.I.I asserts that any actual entity is attached to a space-time region, but the thread of thought is a little clumsy: Whitehead starts with the temporal coordination: *The actual entity is the enjoyment of a certain quantum of physical time,* then he invokes the keyword "standpoint"[11]: *The*

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[8] The notion of a "region" has been used already in UA, but only with respect to space: first as a "portion of space", with the further explanation "not necessarily a continuous portion", but later rather specific for linear or affine subspaces.

[9] More generally, one may consider the regions as the space-time abstractions of events, where an event is *a nexus of actual occasions, inter-related in some determinate fashion in one extensive quantum* (PR 113) – here the last four words *"in one extensive quantum"* should be emphasised.

[10] The difference between „event" and „region" will be discussed later in more detail. Here we quote Palter (1960) who writes: *It seems clear that Whitehead intends regions (the relata of extensive connection) to be formally almost identical with events (the relata of extensions). It is impossible to demonstrate this formal identity between regions and events because Whitehead never lays down a complete set of axioms for either concepts.* (p.109) and: *The sole formal difference between regions and events which is explicitly mentioned by Whitehead, is the fact that regions are limited in extend, or bounded, whereas events may be (as in the case of durations) unbounded.* (p.110)

[11] The pair of notions „standpoint" and „perspective" has to be considered as a unit. In the philosophical tradition, it describes the relationship between the perceiving subject and its perceptions, see already Boethius, but in particular the Monodology of Leibniz (§ 57, and § 60). This pair is used quite often by Whitehead. Chapter II of Part II (with the relevant title *The Extensive Continuum* starts with considerations such as *The world of contemporary actual entities ... is objectified for us as **"realitas objectiva"**, illustrating bare extension with its various parts discriminated by differences of sense data. ... Our direct perception of the contemporary world is thus reduced to extension, defining (i) our own geometrical perspectives, and (ii) possibilities of mutual perspective or other*

quantum is that standpoint in the extensive continuum which is consonant with the subjective aim in its original derivation from God. he invokes But one has to wait until the next paragraph for the coordination in space: *There is a spatial element in the quantum as well as a temporal element. Thus the quantum is an extensive region.* Here we get what we were looking for: the actual entity is assigned a quantum and this is a region in space-time. The formulation “quantum” seems to stress the unity or uniformity of the corresponding region. Of course, subsequently, such a region can be analysed, and thus is divisible, but in itself it is undivided.

The assignment of a region to an actual entity (or a nexus): This assuagement is said to be “blind” (PR 440), since it concerns only the space-time features of the atoms, but not any interpretation or valuation (it is the blindness which does not differ between billiard balls and bullets, between sinus curves and Beethoven’s music, between bits and the information transmitted). To quote Whitehead again, when he refers to actual entities and nexuus*: Both types are correlated by their common extensiveness* (PR 439). But it seems difficult to trace more properties of this assigment, whereas this concerns an important topic. As we have mentioned, this problem is newly-created in PR, since the earlier presentations deal directly with events instead of the now introduced regions. A rather innocent reason for the change of terminology could be the axiomatic approach in PR (so that the different wordings correspond to the different levels), but still the questions remain. In particular, let us repeat: Is an actual entity (or a nexuus) uniquely determined by its space-time extension? What about schizophrenia? Or the difference between the two cultures at a given university? Or the x-ray of a thorax: one event – or two, but the same region. The second question concerns the surjectivity[12]: starting with any

contemporary entities ***inter se****, and (iii) possibilities for division.* (PR 93f). See also: *An act of experience has an objective scheme of extensive order by reason of the double fact that its own perspective standpoint of an actual entity has extensive content, and that the other actual entities are objectified with the retention of their extensive relationships.* (PR 105). Finally, let us quote PR 321 (in Chapter X with the title *Process*): *Objectification is an operation of ... abstraction. ... This fact ... is sometimes termed the perspective of the actual world from the standpoint of that concrescence. Each actual occasion defines its own actual world from which it originates.* Note that there is the additional assertion: *No two occasions can have identical actual worlds.* If we are allowed to replace the words „*actual worlds“* by „regions“, then this provides a confirmation for that the assigment of a region to an actual entity has to be seen as an injective map.

[12] Also the commentaries do not help. One of them, Christian (1959), could be praised for formulating very clear questions – but some of the answers given are quite absurd. He formulates the following theses:

1. An actual entity is extensive.
2. The region of an actual occasion is definite.
3. The region of actual occasions form an extensive plenum.

region – is this the region of an actual entity (or at least a nexus)? The discussion of section IV will yield more insight into the structure of Whitehead's regions.

Connectivity. As we have mentioned, there are two basic notions, that of a region and connectivity. The search for a physical interpretation for connectivity is even more difficult. There are the two complementary concepts of "overlap" and of

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4. No two actual occasions have the same region.
5. The regions of any actual occasions are non-overlapping.
6. Not all the regions that are relata for extensive connection are regions of actual occasions.

(see p.77 for the first five formulation, the last one is mentioned on p.89). Now the first three assertions are unquestionable, and the assertions 4 and 6 have been discussed already. Surprising, and in sharp contrast to Whitehead's text, is thesis 5, since without overlapping and without inclusion (note that inclusions are special cases of overlaps, thus according to Christian also excluded), there cannot be any extensive abstraction – one of the key themes of Whitehead, in particular also of PR. In contrast to the earlier books, the extensive abstraction as presented in PR concerns properly space-time (whereas in PNK and in CN Whitehead tried to isolate the time component first), but this seems to be the only deviation. Christian uses the insufficiently described relationship between events and regions in order to claim a complete break between the conceptions: *... in the earlier writings* ***events*** *not only* ***may*** *include or extend over other events, they* ***always do*** *include some other events and are included by still other events. ... Therefore a proposition analogous to proposition 5, framed in terms of* ***events*** *as described in the earlier writings, would be clearly false.* (p.93/94). As a justification of his interpretation he states: *It is not* ***required*** *by the general theory of extensions that all relations defined in that theory apply to the regions of actual entities* ... In our opinion, such a change of concepts would have been stressed by Whitehead! The theory presented by Christian, concerning actual and possible standpoints (with overlaps occurring only for the possible ones) has no roots in Whitehead's text. What he obtains in this way is a world of entities lying one besides the other, just touching each other and forming a kind of tiling of the universe – in contrast to Whitehead's explicit assertion that the view of a covering by non-overlapping units is just a "logical construct" (PR 508).

It should be added that Palter also wants to see a similar change of concepts (*very nearly a reversal*): *in his early writings there is no doctrine of minimum events; in fact the method of extensive abstraction there explicitly repudiates the idea of either minimum or maximum events.* (p.112) We wonder where he finds "minimal events" in PR! A possible reason for these tiling interpretations seem to be pictorial illustrations as shown in Palter (p.142), which show an actual entity as the crossing square of a duration and a strain. These kinds of illustrations have indeed been used by Whitehead, see Hocking's notes of the Harvard lectures 1924-25 (appendix 1 of Ford, p.282-285), but note that these are local, not global presentations. In this connection, Palter also uses a quote from AI which considers neighbouring relations between actual entities – but he does not take into account the context of the quote: It belongs to a part entitled *The Grouping of Occasions* and concerns a selection actual entities, not the much more involved complete system.

"external connection" and both can be reduced to the inclusion relation. Section IV.I.II (p.436-438) provides some explanation for this, since Whitehead speaks explicitly about "subregions"[13]. In addition, inclusions are considered when he deals with *the perspective of one subregion from the other* (PR 440f). And there are some notes concerning the external relationship at least of ovals, see IV.III.IV (p.468-470), but there under the assumption of a contact surface in space. This discussion concerns the continuous transfer of energy, using the following neighbouring relation*: Let two actual occasions be termed "contiguous" when the regions constituting their "standpoints" are externally connected.* (p.469).[14] Note that often the external connection is implicitly assumed to be temporal, between *antecedent actual occasion, later actual occasion.*

Verification of the axioms. If a system of axioms is given and a presumptive model for it (as in the case we consider), one has to verify that the assumptions hold. This is the point where one would like to see a minimal set of axioms, but this is not known. Thus one would need to check all 20 assumptions – was this ever tried? Where could problems arise? For example, for assumption 2: There are two partial assertions, none seems to be obvious: First, there is the assertion that *no region is connected with all other regions*. And the second: *Any two regions are mediately connected* (in the strong sense of definition 1: using just one intermediate region!). Note also part of the assumption 9*: Every region includes a pair of regions which are not connected with each other* (a kind of separation axiom). Such assumptions may look rather innocent at first glance, but seem to be quite restrictive: several topologies considered by mathematicians, when dealing with spaces arising in nature, do not satisfy such requirements[15]. On the other hand, we should also note the following: several assumptions deal with the existence of subregions. However, none asserts that any region A contains a region B which is non-tangentially included in A (something which seems to be desperately needed in section IV.II.III). Another desiderata: inclusions are needed in order to obtain refinements, and we deal with regions which are (at least) four-dimensional. Thus one needs refinements **in all possible directions:** after all Whitehead wants abstractive elements which converge to points.

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[13] A terminology which is not at all used in Chapters IV.II and IV.III.

[14] It seems that the reader is required to know before-hand what *externally connected* could mean.

[15] As a typical example, take the Zariski topology which plays a decisive role in algebraic geometry.

3 Extensive Abstraction (IV.II.III).

Starting point of the considerations is the conviction that all actual entities are extended in space and in time: that the points in space and in time which are used in the mathematical description of scientific data are obtained by a process of abstraction. The purpose of the method of extensive abstraction is to recover this process.

3.1 Examples.

Let us discuss some examples in detail. We start with the **death of Caesar**: what first comes to mind is *all nature within the Roman senate house during the death of Julius Caesar* (TSM, p.59), but of course one will focus the attention to Caesar himself and to Brutus, to the knife entering the body ... But even if one tries to localise the event in space and in time, using smaller and smaller units, one will obtain a precise time-point only as a limit. Or consider a **lightning**: is it instantaneous? Of course not! One may be able to encircle the event in space as in time, to sharpen the focus, but still there will be duration and spatial extension.

The language used to describe such events is often misleading, at least for scientifically trained people, who are used to deal with functions which provide specific values for points in space-time. But this is the abstraction: the actual event needs *time-duration as well as ... its full spatial dimension* (RM, p.91)[16]. Finally, let us remark that the idea of *simple location* is criticised by Whitehead also in SMW as *mistaking the abstract for the concrete.*

Refinement of perception is very common, in daily life as well as in the history of science. There is the use of eyeglasses, of telescopes and microscopes, the magnification of pictures. We are now used to bits and bytes, to grinds and pixels, to digital data, thus to refinement processes which have a final target: smallest units which cannot be divided further. But note that this depends just on the respective industrial standard, the pixels of a picture may be derived from some higher resolution photo, the music may be remastered by 20-bit technique and so on.[17]

3.2 Abstractive sets

We will not repeat the formal definitions of an abstractive set and its related notions. It may be sufficient to point out that here one deals with a sequence (or set) of nested regions. Let us recall that a classical way for constructing the set of real

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[16] Note that some interpretors are unhappy about that, for example Ross: *It is interesting that Whitehead never considers the posibility that standpoints may not at all be extensive in even a generalized sense* (p.179).

[17] For a general discussion of the measurement of perception, we may refer to R. Efron: The Measurement of Perceptual Durations. In: Fraser-Haber-Müller (1972), p. 207 ff.

numbers starting from the rationals is to consider sequences of nested intervals with lengths converging to zero. Whitehead uses the same recipe in higher dimension (say in dimension 4, in case we consider the regions as entities in space-time, or in arbitrarily large dimension, if we consider events with the full information they carry). Whereas in the one-dimensional case the sequences of nested intervals produce the points on the real line (or better: they "are" representatives of these points), the higher-dimensional analogues produce the points of space-time, but also segments of lines, of surfaces, and so on[18]. If we could interpret regions as sets of points, then we could just take the corresponding set-theoretical intersections[19]. However, this can be done only as an after-thought as soon as points become available. As we know, only via the method of extensive abstraction we may deal with points. It is definition 21 which yields a set of points P(A), for any given region A: the set of points *situated in the region*.[20]

The covering relation. The main way for comparing different abstractive sets is the covering relation (introduced in definition 11): it yields an incidence relation for the corresponding geometrical elements and, equivalently, the inclusion relation for the corresponding point-sets.

Prime geometrical elements. Starting with definition 16, Whitehead discusses properties of abstractive sets which are invariant under equivalence. There is a long-standing tradition in mathematics to try to build up objects from smallest units – they are called primes or irreducibles or indecomposables (the number theory of the integers and related rings considers prime numbers and prime ideals, in algebraic geometry one writes algebraic sets as union of a finite number of irreducible sets). This is the background of Whitehead's considerations concerning prime geometrical elements.

[18] To be precise, we have to stress that different abstractive sets may produce the same geometrical element, in the same way as different sequences of nestes intervalls may converge to the same point. The equivalence relation needed here is introduced in definition 12. Palter has stressed that this means that the so defined geometrical elements (in particular, for example, points) are highly complicated entities, namely equivalence classes of sequences of regions.

[19] It should be mentioned that for any pair of regions belonging to an abstractive set, one of them is non-tangentially included in the other. This requirement is made in order to enforces that the intersections considered may not be empty (even if the point set P(A) corresponding to a given region A is considered as an open set).

[20] The assumptions 29-31 are very restrictive and imply that the geometrical elements constructed by means of extensive abstraction have quite special shapes: a circle cannot be constructed, in contrast to circular arcs.

3.3 Comparison of different versions of extensive abstraction

A short comparison of the different versions of the method of extensive abstraction in the books PNK, CN and PR, as well as de Laguna’s approach, seems to be necessary. Let us start with PNK and CN, where Whitehead uses a two-fold procedure, considering first the time, then the space. The first procedure aims at an isolation of a time coordinate: he wants to construct a duration (but still with a kind of extension in time). Abstractive sets are used, working with unbounded sets. This has been strongly criticised by de Laguna. The second procedure then uses the method of extensive abstraction, starting with events. Here Whitehead insists that one should work with bounded regions (or better, with *limited events*). The presentation in CN is less formal compared to PR, there is no explicit mentioning of definitions, assumptions or proofs, but otherwise quite similar, and Whitehead acknowledges in PR those deviations which he feels are essential. He also stresses that the axiomatic approach given in PR is influenced by de Laguna. In particular, he gives credit to de Laguna for the idea to replace the concept of “inclusion” by “connectivity“, and there he distinguishes between “overlapping” and “external connectedness”. Note that the connectivity theory allows to consider tangentially as well as non-tangentially inclusions. Stimulated by the earlier approaches of Whitehead, de Laguna himself has put forward a theory of space: he restricts to spatial extensions and starts with bounded 3-dimensional regions.

Now to PR: First of all, the former attempt to separate time and space has been abandoned – a very important and prospective decision. But there is also the more technical change which has already been mentioned: taking into account objections of de Laguna, Whitehead allows only non-tangentially inclusion when dealing with abstractive sets. What is the difference, and what kind of imagination is lying behind it? Whitehead wants an abstractive set to be seen as to converge to a set of inner points – to points which are relevant to all the regions involved. If we would allow tangentially inclusions, then the abstractive set may converge to boundary points. Also, abstractive sets without any overlaps may converge to the same geometrical element. And this is not what Whitehead wants: the convergence procedure should yield as a result geometrical elements which are inherent in the regions involved.

3.4 Other approaches

As we have outlined, Whitehead’s method of extensive abstraction generalises the construction of the real numbers by using sequences of nested intervals. Other possibilities for creating the real numbers are known. The Dedekind cuts deal with unbounded subsets of the rational numbers (in some sense this may be compared with the introduction of durations in PNK and CN). And there are the Cauchysequences, but such an approach would be alien to Whitehead's setting, since it requires that some space-time points are already given.

Given any topological space T [21], the knowledge of the "open" sets allows to reconstruct the points of the total space, provided some separation axiom (the so-called Frechet axiom) is satisfied: then the points correspond to the minimal non-empty closed subsets[22].

The idea to work with topological spaces not taking into account points but dealing only with the system of what should be the open sets is usually attributed to Lesniewski[23]. There is the famous article by Menger: *Topology without points* (1940), and many other mathematicians have dealt with this setting which is usually referred to as "mereology" (see for example the survey by Peter Simons (1991)).

We should also mention the Proceedings of a 1969 Oberwolfach conference with the title *The study of time* (edited by Fraser, Haber, Müller (1972)); several of the contributions[24] discuss the relationship between instants in time and time intervals.

3.5 Grünbaums's criticism

Grünbaum has considered Whitehead's theory of extension several times. In particular let us look at his 1953 paper in the British Journal of Philosophy, and his 1962 review of the book of Palter. Beth (1954) summarises Grünbaum's objections as follows: (i) *Even if the existence of denumerable actual infinite is somehow certifiable by sense awareness, sense awareness cannot suggest the idea of a super-denumerable collection of perceptible regions, which is needed in order to avoid Zeno's paradox of plurality*, (ii) *that the convergence of Whitehead's classes is am-*

[21] This means: a set (the "total space") is given, together with a set of distinguished subsets satisfying suitable axioms; the distinguished subsets are called "open" sets, their complements are the "closed" sets.

[22] If one wants to obtain the points as the intersection of a countable sequence of open sets, then a further axiom, the first countability axiom, is required.

[23] He developed this idea in his book "Foundation of general set theory I" (Moskau 1916) and in "Foundation of Mathematics" (1927 – 1931).

[24] A.N.Prior: *The Notion of the Present*, C.L.Hamblin: *Instants and Intervals*, M. v. Capek: *The Fiction of Instants*. E.Cassirer: *On the Reality of Becoming*. W. Mays: *Whitehead and the Philosophy of Time*. Not only Mays, but also v. Capek explicitly mention Whitehead. Of interest seems to be the paper by Hamblin – not only with respect to his final remark: *Is was drawn to my attention during the Oberwolfach conference by H.A.C. Dobbs that the definition of instants from intervals in set-theoretical terms has previously been discussed by A.G. Walker in* [7] (p.331) . The paper [7] has the *title Durees et instants* and has appeared in 1947 in "Revue des cours scientifiques". Maybe Hamblin should have looked at Whitehead!

biguous, and (iii) *that these classes do not belong to the domain of sense awareness.* With respect to the last two assertions, we feel that there is no real dissent to what Whitehead writes. But it seems necessary to discuss in detail the first argument, the comprehension of infinite sets of data. We first should recall the different levels of infinity, at least the distinction between denumerable and super-denumerable infinite sets as introduced by Cantor, since Grünbaum's objection is based on this difference[25].

Many parts of mathematics (for example analysis) deal with sets which are infinite. Since Cantor it is customary to distinguish infinite sets according to their cardinality: Two sets S, S' are said to have the same cardinality provided there exists a bijection between the elements of S and of S'; a set S is said to be "denumerable" (or to have cardinality aleph zero) provided it has the same cardinality as the set **N** of all natural numbers, and "super-denumerable" provided it is infinite and not denumerable. An easy argument shows that the power set[26] **P**(S) of a set S never has the same cardinality as S, this shows that **P**(**N**) is super-denumerable. Also, it is easy to see that the set **R** of all real numbers is super-denumerable, the same is true already for any interval [a,b]. On the other hand, the set **Q** of all rational numbers is denumerable – this seems to be surprising: after all, **Q** is a dense subset of **R**.

Let us return to Grünbaum, who stresses the following: *Empiricists from Aristotle to Hume have maintained that* ***infinitum actu non datur***, but he adds: *Let us suppose for the argument that contrary to that tradition, the existence of a* ***denumerable*** *actual infinite were somehow certifiable by sense awareness so that the meaning of aleph zero could still be given a sensationalist pedigree. It would then nevertheless be true that the very notion of actually infinite classes having a cardinality* ***exceeding*** *aleph zero would inexorably defy encompassment by the sensory imagination. For the set-theoretical meaning of super-denumerability eludes all logically possible* ***sensory*** *exemplification, since* ***any*** *collection of non-overlapping three-dimensional regions of space is at most* ***denumerably*** *infinite (cf. G. Cantor, Math. Ann.1882, 20, 117).* In order to analyse these considerations, let us begin with the last argument starting with "since". Cantor's assertion is correct, but has nothing to do with the problem considered here. Actually, it helps to understand the situation! The deceptive word used is "non-overlapping": Clearly, the topological spaces which are of interest here (such as **R** or the usual manifolds considered

[25] In his review Beth also writes: *Though the author's conclusions are probably correct, argument* (i) *does not seem fully convincing on account of the Skolem-Lowenheim paradox*. This paradox asserts that in elementary logic, references to super-denumerable sets are not really daunting, since any given model can be replaced by a similar one which uses only denumerable sets. We do not see in which way a reference to the Skolem-Lowenheim paradox is really relevant for the problem in question.

[26] This is the set of all subsets of S.

in cosmology) have a denumerable topological basis, so that any family of pairwise disjoint open sets has to be at most denumerable. The super-denumerabilty of **R** (and of corresponding manifolds) has nothing to do with the global structure of **R**, but is a purely local phenomenon: For example, it is easy to see that **R** can be covered by a denumerable number of bounded intervals, say the intervals [n,n+1], and it is each of these intervals which is super-denumerable!

The reader should be reminded that this argument is supposed to criticise Whitehead's method of extensive abstraction. But looking at this method, one observes that super-denumerable sets of regions are never used! On the contrary, Whitehead's abstractive sets are denumerable sequences, and it will even be sufficient to assume that one starts with only a denumerable set of regions (recall that the nested interval construction of **R** starts with intervals with rational boundary numbers, thus with a denumerable set of intervals).

Now we could end the discussion, but we use the opportunity to scrutinise what finite determination of sensory imagination could mean. Recall that according to Grünbaum, the only data which may be certifiable by sense awareness are finite or denumerably infinite ones. We strongly disagree! It seems obvious that anyone is able to "see" an interval such as [0,1], without even being aware that mathematicians would characterise this as a typical super-denumerable set. In many respects, this super-denumerable interval is easier to visualise than the subset of all the rational numbers in [0,1] (and this subset is denumerable). It may be reasonable here to draw the attention to computer graphic programs and the difference between pixel description and vector graphics, or to the general problem of digitalisation of data. The question to describe the finite nature of sense data is an important one, but has nothing to do with the denumerability.

A further question has to be added: if one assumes the finite nature of sense data, one may ask in which way and to what extend refinements are possible. Of course, here we are back at the process of extensive abstraction! Now Grünbaum asserts that extensive abstraction is not a sense datum. But Whitehead himself writes*: ... the restless modern search for increased accuracy of observation and for increased detailed explanation is based upon unquestioning faith in the reign of Law. Apart from such faith, the enterprise of science is foolish, hopeless.* (AI, p.135). Here is another quote: *The method is merely the systematisation of the instinctive procedure of habitual experience.* (PNK, p.76).

It seems to be of interest to see what kind of alternative Grünbaum may have in mind when he criticises Whitehead. Note that he does **not** object the use of real numbers in mathematics or physics when describing space-time phenomena. In the same way as Whitehead, he considers the set **R** of real numbers as an abstraction, but he just conceals the way to obtain them. Any construction of the real numbers has to be based on some sort of denumerable convergence. Since one has to obtain not only the points but also a corresponding topology, Whitehead's mereological approach seems to be most efficient.

3.6 The boundary of a region.

We have mentioned already that definition 21 allows to attach to any region A a point set P(A), namely the set of *points situated in the region*; Whitehead calls it the *volume* of the region.

Similarly, the definition 22 attaches to the region A its boundary O(A): this is the set of all points x which are not situated in A, but such that any region B with x situated in B overlaps A; Whitehead calls it the *surface* of the region.

It seems to be necessary to have a detailed look at the assumptions 29 – 31. They are usually not taken into account[27], but show very clearly the strict restrictions for what Whitehead allows to be called a "region". What is asserted here? First of all, that the sets P(A) correspond bijectively to the regions A. In particular, this implies that for any region A, there have to exist regions B with B non-tangentially included into A (since A must belong to some abstractive set). Secondly: also the sets O(A) correspond bijectively to the regions A: every region is uniquely determined by its boundary. These seem to be the relevant parts of assumption 29. Assumption 30 then asserts that P(A) is path-connected. Correspondingly, assumption 31 yields the same assertion for O(A), namely that also the boundary of a region is path-connected. These assumptions suggest that Whitehead wants that for any region A, the set P(A) to be open in its closure – a quite reasonable wish. But we also see that Whitehead feels, that regions should not have internal holes (the existence of internal holes in A would contradict the connectivity of O(A)). Another problem has to be mentioned: The assertion that O(A) uniquely determines A sounds rather innocent, but actually it is a very strong global restriction! In order to see this, consider the two-dimensional analogue setting of a sphere and take as regions just circular discs. For large circular discs, say with boundary a great circle, the boundary no longer determines its interior.

It seems that Whitehead never tried to provide any explanation for assumption 29, indeed it even seems that this assumption does not correspond at all to his clearly formulated rejection of punctual determination! In our interpretation, assumption 29 wants to assert that punctual constructions (abstractions) provide well-formed shapes: If the regions have well-formed boundaries (and assumption 29 has to be

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[27] On the contrary, Palter (1960) for example claims that such considerations are missing in PR! He writes: *Whitehead may wish to exclude regions with "holes" as he excludes events with "holes" in his earlier works; but this is by no means certain, since his later theory of extensions* (and here Palter means PR) *is deliberately more general than his earlier theory* (p.146).

read in this way), then one can use these boundaries in order to develop an exact calculus. Note that modern mathematics provides several approaches in order to avoid the use of well-formed boundaries: to neglect sets of measure zero, to work with stochastic differential equations, to deal with fractal boundaries, or see the fuzzy set theory. Already the formulation of the axioms of topology, using neighbourhood systems instead of open sets, is a first attempt to put aside the structure of boundaries. The possibility of rough boundaries has to be seen as one of the main features of a Whiteheadian cosmology. There is a corresponding formulation of Whitehead himself: *Events appear as indefinite entities without clear demarcations* (PNK 73), see also (CN 59).

4 Flat Loci (IV.III)

This chapter contains a lot of considerations which are quite obsolete by now. Whitehead tries to describe a kind of differential structure on the space-time manifold in terms of set-theoretical topology. This could be of value. However, his insistence on flatness should remind the reader on all the vain attempts to prove that earth is a disk ...

Palter[28] reports that Whitehead tried to convince Einstein that space-time cannot be curved, for philosophical reasons! For example, Whitehead wants that any two points in space time are connected by a uniquely determined line.

There are good reasons that this chapter usually is not mentioned at all in the literature. But some of the ideas may still be of interest.

4.1 Section IV.III.I.

This section has to be seen as a bridge passage. Its last paragraph provides the final touch to the considerations of Chapter IV.II, specifying again the ontological character of a point (or any other geometrical element obtained by extensive abstraction): it is a nexus of actual entities. This comes not as a surprise, since it just articulates the procedure of obtaining geometrical elements: as a set of regions with specified properties. We are just told that geometrical elements are what they are.

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[28] *Whitehead had long discussions with Einstein and repeatedly attempted to convince him that on metaphysical grounds the attempt must be made to get along without the assumption of a curvature of space. Einstein, however, was not inclined to give up a theory, against which neither logical nor experimental reasons could be cited, nor considerations of simplicity and beauty. Whitehead's metaphysics did not seem quite plausible to him.* (P.Frank: Einstein, His Life and Times, p.189)

4.2 Sections IV.III.II and IV.III.III.

We have seen in IV.II.III that for any pair of points, there are a lot of segments with these endpoints. Now Whitehead wants to single out a unique such segments which he calls *linear*: it is supposed to be straight and flat.

In order to do so, he introduces the concept of an *ovate class of regions*. By definition, this is a subset of the set of all regions with very concise intersection properties (both with respect to intersections among each other, as well as for intersections with arbitrary regions). If an ovate class of region is given, its elements are called *ovals* [29]. Then, in section IV.III.III, the first assumption postulates the existence of such an ovate class (with the somewhat strange addendum "in the extensive continuum of the present epoch"). Whitehead guesses *(it seems probable*, PR 462) that there should be only one such class.

If one considers the n-dimensional real space $\mathbf{R}^n$ (and clearly it is this space which is the standard model for dealing with extensive connection), then the set of convex open subsets is such an ovate class of regions. Let us recall that a subset of $\mathbf{R}^n$ is said to be convex, provided it contains with every pair of points a,b also the line from a to b.

Some of the conditions on an ovate class concern the existence of abstractive sets which contain only ovals. This set of conditions is thus called the abstractive group of axioms. We will not discuss these axioms in detail, but let us draw the attention to assumption 2 (PR 465). It seems to be of great importance for Whitehead and he includes a formal proof in the style of a mathematical text. Assumption 2 asserts that certain abstractive sets of ovals are equivalent (thus they yield the same geometrical elements), namely those which are prime with respect to covering a fixed set $P_1,...,P_m$ of points. In case m = 2, one will obtain in this way a straight segment or straight line with endpoints P_1 and P_2, for m = 3 a triangle, for m = 4 a tetrahedron (definitions 5,7,9). The further definitions provide the corresponding global notions (of a line, a plane, a flat 3-dimensional subspace, respectively). Here, a warning seems to be necessary: Let us consider the analogue 2-dimensional case of a 2-sphere S (in contrast to the flat 2-dimensional real space $\mathbf{R}^2$). Given a pair of points P_1 and P_2 on S, there will exist a unique shortest path between P_1 and P_2 on S only in case the points are not antipodes! In case P_1 and P_2 are antipodes, there is a whole family of shortest paths between these points (all being halves of great circles) and one of them can be distinguished by a characteristic property. This already indicates that the assumption that an ovate class of region exists, presupposes some flatness hypothesis.

[29] Note the following: the decision whether or not a given region A is an oval cannot be made by looking at A alone – one needs to know the complete ovate class.

4.3 Section IV.III.IV.

This section contains some explanations concerning the concept of external connection. First, Whitehead deals with the problem in which way different regions may touch each other: the discussion is restricted to ovals, and deals mainly with contact in time (*the objectification of the antecedent occasion in the later occasion*, PR 468, the corresponding actual entities being called *contiguous*). The further parts of this section as well as all of section IV.III.V is devoted to the cosmological interpretation which was missing until now. For the division of space-time in space and time coordinates, one has to wait until Chapter IV.IV, a chapter which is devoted to the so-called *strains*. But at least the contact in time is discussed at this stage. Such a contact is related to the transfer, say, of energy or information. Whitehead does not want to exclude the possibility of distant effects, but stresses that there seems to be a lot of evidence that all the forces function via direct contact along a sequence of intermediate regions*: through a route of successive quanta of extensiveness. These quanta of extensiveness are the basic regions of successive contiguous occasions* (PR 468).

Unfortunately, the further distinction *between immediate objectification for the mental pole and the mediate objectification for the physical pole* sounds quite mystical. But perhaps the following interpretation may help: The prehension of encoded information (language, music, ...) allows to jump over neighbouring entities, and thus yields an immediate objectification in a situation in which otherwise only mediate objectification would be possible.

4.4 Vector fields

Let us focus the attention to the final two paragraphs of section IV.III.V, which highlight a new keyword, that of a vector. As we have mentioned before, in order to understand the references to vectors in the framework of Whitehead's philosophy, it is not sufficient to have a single vectorspace and its elements (the vectors) in mind. But one has to envision vector fields as they arise say in the theory of differential equations, or when dealing with dynamical systems. If one agrees that Chapter IV.III tries to present an approach to cosmology in the spirit of what now would be called differential geometry, then our interpretation fits this intention very well. But note that differential geometry and vector fields are explicitly mentioned at the end of Chapter IV.IV (PR 507) only.

Vector fields are something very natural and very basic: one attaches to every point of a manifold a vector (thus a direction and a number, namely the length of the vector); everybody is familiar with such a presentation, say looking at a weather chart with the vectors indicating the wind direction and its force, or looking at a marine chart indicating the current, and so on. Unfortunately, the mathematics needed to deal with them is somewhat intricate, thus they are not commonly included in the ordinary school curriculum. Actually, such vector fields are discussed in high school, but only outside of mathematics, say in geography, in physics (for example: magnetic fields), or in biology. Vector fields are by now one of the most important tools for a mathematical description of processes as they are considered in science as well as in economy. Mathematical models of dynamical systems use differential equations and the corresponding phase diagrams. One of the aims of such a presentation is to provide predictions (for the weather, for shares and bonds, for the gravitational force, ...) Note that such differential equation models are based on the assumption that the corresponding forces are of local nature.

It should be stressed that Whitehead's theory of prehension, as presented in Part III of PR, has to be interpreted in terms of vector fields, too. All the interactions between actual entities, which is discussed in the genetic analysis, have to be seen in this way.

4.5 Section IV.III.V: Recapitulation

The first four paragraphs indicate the position of the theory of extension in the full context of *Process and Reality*, thus it seems worthwhile to look at them in detail. Whitehead stresses the relational character of his theory. In contrast to the Cartesian view of physical bodies and their attributes, Whitehead insists on the fundamental importance of the relations *within* actual entities and *between* them. These relations are described in terms of the theory of extension, those within an actual entity via the notion of inclusion, and those between actual entities via overlap and via external connectivity. Thus we see that it is the topology of the set of regions which is the basis of the organistic philosophy of Whitehead. One finds a more detailed description of these two kinds of relations elsewhere: For the relation of external connection (at least in respect to time) one has to refer to Chapter IV.IV (*strains*), whereas the inclusion scheme, which is relevant for the genetic process has been discussed in Part III (*The Theory of Prehension*).

The last two paragraphs argue against the division between matter and empty space. The vector interpretation is based on the importance of action and flow, Heraklit's dictum "everything flows" is translated into the formulation *all things are vectors*. Note that in this way the flow has a kind of quantum characteristic. Flow has to be considered as a nexus of actual entities, namely a nexus of successive actual entities.

4.6 Actual entities

The actual entities have to be seen as the final units, and it is of importance to accept that they have an extension in space-time. There is a final footnote in Chapter IV which seems to be a source for great confusion: Whitehead calls his theory a *doctrine of "microscopic atomic occasions"* (PR 508)[30]. The reader may wonder why the actual entities are not just labelled *atomic*, but *microscopic atomic*. Where do we find hints about the use of the word "microscopic"? The index lists the pages 75, 196, 254, 326, 327 – all concern the history of philosophy, whereas the systematic parts (with the exception of this final footnote of Part IV) do not invoke such an idea. Let us see in which way the relationship between macroscopic and microscopic view is discussed (see PR 75): Whitehead considers the process of concrescence, and here he distinguishes between the initial status or facts (the macroscopic view) and the final status or facts (the microscopic view). The subjective unity of the actual entity (and this is the final fact, thus the microscopic view) requires to see the concrescence, the standpoint of the actual entity (its region) as a unit, as a quantum. As Whitehead often writes*: divisible, but undivided*.

The footnote in question concerns a dispute with Northrop, thus it is necessary to consult his corresponding texts. But a translation of what Northrop calls a macroscopic atom into the categorical scheme of Whitehead provides clarification: Northrop considers a person and its development in time, or the solar system or also a molecule, always with all their changes ... But in the terminology of Whitehead, these are not actual entities, but nexuus[31] of actual entities! The identification of a person over the years has to be considered as an idea, an eternal object, but not as reality.

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[PNK] The Principles of Natural Knowledge. 1919.
[TSM] Time, Space, and Material: Are they, and if in what Sense, the Ultimate Data of Science? Proceedings of the Aristotelian Society, Suppl. 2, 44-57.
[CN] Concept of Nature. 1920.
[REL] The Principle of Relativity with applications of Physical Science. 1922.
[SMW] Science and the Modern World. 1925.
[PR] Process and Reality. 1929. (The page numbers refer to the Harper Torchbooks edition 1960.)
[AI] Adventures of Ideas. 1933.

References 2: Comments.

[30] In a first reading, this remark must be seen as a bomb which may destroy the whole categorical scheme! And note that it occurs on the final lines of Part IV (and thereafter, there is only God).

[31] It should be noted that Whitehead himself sometimes is tempted to smear the diefference between such nexuus and the actual entities themselves, see for example PR 439.

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