Summer term 2023
Prof. Dr. Michael Röckner and Sebastian Grube

Exercises to Introduction to Stochastic Partial Differential Equations I

Sheet 7
Total points: 14
Submission before: Friday, 19.05.2023, 12:00 noon

Problem 1 (Prove the details). (1+1+1+1+2 Points)

Consider the proof of Lemma 2.4.3.

(i) (Step 1) Prove that for all $n \in \mathbb{N}$

$$||\tilde{\Phi}_{\zeta_n} - \tilde{\Phi}_{\zeta}||_T \leqslant \sup_{t\in[0,T]} ||\zeta_n(t) - \zeta(t)||\ ||\Phi||_T.$$

(ii) (Step 1) Prove that P-a.s.

$$1_{\bigcup_{m=1}^{\infty}\{\tau_{m-1}<T=\tau_m\}} = 1.$$

(iii) (Step 2) Explain the penultimate equality.

Consider the proof of Lemma 2.4.4.

(v) How *exactly* can Lebesgue's dominated convergence theorem be used to conclude that

$$\sum_{i=1}^{\infty} E\left(\left|\sum_{t_{j+1}^l\leqslant t} \langle e_i, M^{\tau_N}(t_{j+1}^l) - M^{\tau_N}(t_j^l)\rangle_H^2 - \int_0^{t\wedge\tau_N} ||\Phi(s)^* e_i||_{U_0}^2\ ds\right|\right) \to 0,\ l\to\infty.$$

Consider the proof of Proposition 2.5.2.

(vi) Prove in detail that $P \circ (W^J(t) - W^J(s))^{-1} = N(0, (t-s)JJ^*)$.

Problem 2 (Definition (2.5.2)). (4 Points)

Let W be a cylindrical Q-Wiener process. Prove that the stochastic integrals $\int_0^t \Phi(s)dW(s)$, $t\in[0,T]$, can be uniquely defined for every process $\Phi \in N_W$.

Problem 3. (4 Points)

Prove Exercise 2.5.4. in the lecture notes.