Derived invariants of algebras

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Classical Morita theory describes when two algebras have the 'same' representation theory. Quite often it was observed that algebras can share many representation theoretic properties, but without being Morita equivalent. Starting with work of Happel, Rickard and others, many of these observations could be structurally explained by equivalences between derived module categories.

This can be seen as part of a general philosophy of using methods from homological algebra in representation theory. Many homological invariants of algebras are still only partially understood, and there is a long list of open homological conjectures.

In the first part of the talk we survey some recent results on derived equivalence classifications for particular classes of algebras (blocks of group algebras, certain classes of tame algebras).

In the second part we are going to discuss the representation dimension, a homological invariant introduced by M. Auslander. It turns out that the precise value of the representation dimension is difficult to determine, even for 'small' examples. For selfinjective algebras the representation dimension is an invariant of the derived category (C. Xi). Algebras with small representation dimension are of particular interest, due to a result of Igusa and Todorov: if an algebra A has representation dimension at most 3, then the important open finitistic dimension conjecture holds for A. In the talk we present recent work showing that the representation dimension actually is at most 3 for several prominent classes of tame algebras (special biserial algebras, blocks of group algebras and Schur algebras).