

HECKE ALGEBRAS, PERVERSE SHEAVES AND FINITE REDUCTIVE GROUPS

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“Hecke algebras” were first studied by Iwahori in connection with the decomposition of the permutation representation of a reductive group over a finite field on the rational points of its flag variety. Their representation theory is now central to the study of reductive groups over finite and local fields. I shall explain various connections, how Hecke algebras arise in connection with perverse sheaves on Schubert varieties, and how they are connected with the study of the “Green functions” of reductive groups via the geometry of unipotent orbits.

The detailed contents of my lectures are as follows:

1. LECTURE 1

1. Characters of $GL_n(\mathbb{F}_q)$.
2. Harish-Chandra induction and cuspidal characters.
3. Hecke algebras and decomposition of induced cuspidal characters.
4. Kazhdan-Lusztig theory for Hecke algebras.
5. Deligne-Lusztig theory for \mathbf{G}^F (\mathbf{G} a reductive group, F =Frobenius).
6. Green functions: their rôle; some open problems.

2. LECTURE 2

7. Green functions (continued); connections with unipotent orbit geometry.
8. The space $\mathcal{C}_{\text{uni}}(\mathbf{G}^F)$ of unipotently supported class functions on \mathbf{G}^F .
9. Lusztig induction and restriction.
10. A generic (i.e. field independent or geometric) description of $\mathcal{C}_{\text{uni}}(\mathbf{G}^F)$ (joint work with Digne and Michel).
11. Inner product formulae. Connection with Kazhdan-Lusztig polynomials.
12. Open problems.