Tame Frobenius algebras and infinitesimal groups

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From Drozd's Tame and Wild Theorem the class of finite dimensional algebras over an algebraically closed field k may be subdivided into two disjoint classes. One class consists of the tame algebras for which the indecomposable modules occur in each dimension in a finite number of discrete and a finite number of one-parameter families. The second class is formed by the wild algebras whose representation theory comprises the representation theories of all finite dimensional algebras over k. Accordingly, a classification of the indecomposable finite dimensional modules in feasible only for the tame algebras. Among the tame algebras we may distinguish the classes of: representation-finite, domestic and polynomial growth algebras. The representation theory of representation-finite algebras is presently rather well understood while the representation theory of representation-infinite tame algebras is still only emerging.

In the first part of lectures we will give a short overlook on the representation theory of tame (basic) Frobenius algebras. In our considerations, a crucial role will be played by the Frobenius algebras which admit Galois coverings by repetitive algebras.

In the second part of lectures we will be concerned with the determination of tame finite dimensional Hopf algebras $\mathrm{H}(\mathcal{G})$ of infinitesimal groups $\mathcal{G}$, equivalently tame finite dimensional cocommutative Hopf algebras with local dual algebras. The treatment of these algebras necessitates an approach different from the methods employed in the modular representation theory of finite groups. Geometric and homological techniques, combined with methods from abstract representation theory, provide a good understanding of two special cases given by semisimple groups and groups of height 1 (equivalently, restricted Lie algebras). We will show how the structural results concerning these cases in conjunction with interpretation of Galois extensions as Galois coverings allow the determination of infinitesimal groups of odd characteristic whose Hopf algebras possess a tame principal block. Moreover, the structure of all tame blocks of these Hopf algebras will be also determined. We note that in contrary to the modular representation theory of finite groups, the principal block of an arbitrary finite dimensional Hopf algebra is not necessarily the most complicated one.