Quadratic Auslander regular algebras: nontrivial stably free modules and representations

Natalia K.Iyudu

In this talk I intend to consider a class of noncommutative quadratic algebras that have appeared in physics, the so called RIT algebras, from several points of view: I will construct nontrivial stably free modules, derive a series of homological properties from combinatorial ones and investigate some questions concerning their representation theory.

The algebras in question are given by generators and relations. The simplest one is $K < x, y | xy - yx = y^2 >$. We show that these algebras are endowed by an I-type structure. As a consequence we have that they enjoy a series of nice properties (they are Koszul, Cohen-Macauley, Noetherian domains) in particular they are Auslander regular.

For any algebra from this class we construct a stably free non-free module. This gives a counterexample to the noncommutative analogue of Serre's conjecture about freeness of f.g. projective modules.

We consider then more closely the algebra of global dimension two from this class. (According to the classification of Artin and Schelter there are only two Auslander regular algebras: one is our algebra, given by relation $xy - yx = y^2$, another is the usual quantum plane $xy = qyx$.) We classify all n-dimensional representations ρ_n of this algebra using quiver-equivalence. As a consequence we mention meanwhile that our algebra is residually finite-dimensional. Some kind of description of infinite dimensional simple modules over simplest RIT also will be given.