Christof Geiß:
Auslander algebras and initial seeds for cluster algebras

Let Q be a Dynkin quiver and Π the corresponding set of positive roots. For the preprojective algebra Λ associated to Q we produce a rigid Λ-module I_Q with $r = |\Pi|$ pairwise non-isomorphic indecomposable summands by pushing the injective modules of the Auslander algebra of kQ to Λ.

If N is a maximal unipotent subgroup of a complex simply connected simple Lie group of type $|Q|$, then the coordinate ring $\mathbb{C}[N]$ is an upper cluster algebra. We show that the elements of the dual semicanonical basis which correspond to the indecomposable summands of I_Q coincide with certain generalized minors which form an initial cluster for $\mathbb{C}[N]$, and that the corresponding exchange matrix of this cluster can be read from the Gabriel quiver of $\operatorname{End}_\Lambda(I_Q)$.