Bifurcations in Dynamical Systems with Applications

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SFB 701

Department of Mathematics, Bielefeld University

Abstracts

Localization and continuation of nonlinear eigenvalues

Wolf-Jürgen Beyn (Bielefeld)

Nonlinear eigenvalue problems are ubiquitous in the stability analysis of nonlinear systems, such as vibrating systems or systems with delay. Numerical discretizations then lead to large and sparse parameterized nonlinear eigenvalue problems

$$A(s,\lambda)v = 0, \quad v \in \mathbb{C}^m,$$

where the matrix family $A(s,\lambda) \in \mathbb{C}^{m\times m}$ depends smoothly on the real parameter $s \in \mathbb{R}$ and analytically on the eigenvalue parameter $\lambda \in \mathbb{C}$. We aim at an algorithm that detects a small swarm of eigenvalues λ within a prescribed complex domain and that continues the swarm with respect to the parameter s.

A new localization procedure is presented that determines the eigenvalues (and eigenvectors) in the interior of a smooth contour of the complex plane. The method builds on Cauchy's integral formula and on a theorem of Keldysh. Then we discuss a continuation method that pursues the swarm of eigenvalues with the parameter and that deflates and inflates the swarm when collisions with outside eigenvalues occur.

Localised travelling waves in nonlinear lattices

Alan Champneys (Bristol)

In this talk, which represents joint work with my PhD student Tom Melvin, we shall address bifurcations of travelling waves in nonlinear Schrödinger-type lattices. Using a pseudo-spectral method first used by Eilbeck we find localised waves as solutions of advanced-delay equations. The spectrum of the linearised problem provides parameter regions where there is a single phonon band (branch of linear waves). In such regions, paths of localised solutions exist for isolated wavespeeds.

In collaboration with Dimitry Pelinovsky we also find analytically the minimum wavespeeds at which such waves can bifurcate. This can be acheived by either a Melnikov argument from a certain codimension-two point, or by computation of the Stokes constant in a beyond-all-orders limit. In so doing we can explain how some waves can persist as one interpolates between Ablowitz-Ladiq and dNLS lattices.

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An interesting bifurcation scenario in a 3D laser model - with open questions ...

Eusebius Doedel (Montreal)

In recent work with Carlos Pando (Puebla) on a laser model we have found an interesting bifurcation scenario involving (among other things) isolas of periodic solutions. I will give a brief presentation of these results in the hope that it will generate a discussion about the underlying mechanisms and about related work.

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Continuation in reversible systems with application to degenerate subharmonic bifurcations

Jorge Galan-Vioque (Sevilla)

Joint work with A. Vanderbauwhede, E. Freire and F. J. Muñoz-Almaraz.

We report both analytical and numerical results for the degenerate subharmonic bifurcation in reversible systems. In particular we will concentrate in the case in which the Floquet multipliers evolve around the unit circle, stop at a q-root of unity and reverse the sense of rotation. The theory predicts two possible scenarios; namely, two connecting branches (bananas) or two non connected branches (banana split). The branch switching and the numerical continuation is difficult and relies on the reversibility properties of the solution. The presence of additional conserved quantities introduces further degeneracies.

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Bifurcations of maps: algorithms and applications

Willy Govaerts (Gent)

Joint work with Reza Khoshsiar Ghaziani, Yuri A. Kuznetsov, Hil G.E. Meijer.

Dynamical systems come in two forms: continuous or discrete. In this talk we discuss the numerical bifurcation study of discrete-time maps and the implementation of the numerical methods in the package Cl_MatContM (Command line Matlab Continuation for Maps).

The package is basically a continuation package (like AUTO and CONTENT), i.e. its kernel is a continuation routine for the computation of solutions to a system of equations under parameter variation. The system of equations can describe any type of objects, for example $n-$ cycles of the map, or Neimark-Sacker bifurcation points of cycles of maps. The number of free parameters must be one higher than the codimension of the bifurcation, since we need systems where the number of variables is the number of equations plus one.

The package allows to continue cycles of maps and their codimension one bifurcations. During these continuations, all generically possible bifurcations of higher codimension are detected, located and their normal forms are computed. Furthermore, from a codimension 1 or codimension 2 bifurcation point it is possible to start the curves of lower codimension that are generically rooted in such points.

The continuation of normal form coefficients requires accurate derivatives of high order (up to 5). This can be done using symbolic derivatives or automatic differentiation.

Apart from the continuation, the package contains also some other routines, namely for the computation of one-dimensional stable and unstable manifolds in a 2D-system. These are needed to obtain initial guesses for the continuation of homoclinic and heteroclinic connections.

While continuing homoclinic or heteroclinic connections it is possible to detect limit points, which generically correspond to non-transversal intersections of the stable and unstable manifolds in the points of the connection. It is then also possible to continue such limit points in two free parameters.

The software was already applied to models from biology and economics. In several cases it allowed either to extend the understanding of such models or to correct erroneous statements in the existing literature.

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Verifying exponential dichotomies numerically

Thorsten Hüls (Bielefeld)

In this talk, a numerical method is introduced for testing whether a linear difference equation possesses an exponential dichotomy. If this is the case, we approximate dichotomy rates as well as the corresponding projectors with high accuracy. The suggested approach is based on solving linear boundary value problems, having a sparse structure. For an illustration, we use Hénon's map and approximate dichotomy rates and projectors of the variational equation along a homoclinic orbit and an orbit on the attractor. Approximation errors that occur, when restricting the infinite dimensional problem to a finite interval, are analyzed in detail.

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Discretization of transfer operators using a sparse hierarchical tensor basis-the Sparse Ulam method

Oliver Junge (München)

The global macroscopic behaviour of a dynamical system is encoded in the eigenfunctions of a certain transfer operator associated to it. For systems with low dimensional long term dynamics, efficient techniques exist for a numerical approximation of the most important eigenfunctions. They are based on a projection of the operator onto a space of piecewise constant functions supported on a neighborhood of the attractor - Ulam's method.

In this talk we develop a numerical technique which makes Ulam's approach applicable to systems with higher dimensional long term dynamics. It is based on ideas for the treatment of higher dimensional partial differential equations using sparse grids. We establish statements about its complexity and convergence properties and present two numerical examples.

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Bifurcations and continuous transitions of attractors in autonomous and nonautonomous systems

Peter Kloeden (Frankfurt am Main)

Nonautonomous bifurcation theory studies the change of attractors of nonautonomous systems which are introduced here with the process formalism as well as the skew product formalism.

We present a total stability theorem ensuring the existence of nearby attractors of perturbed systems. They depend continuously on a parameter if and only if the attraction is uniform w.r.t. parameter, i.e. the attractors are equi-attracting.

We apply these principles to explicit systems to clarify the meaning of continuous and abrupt transitions of attractors in contrast to bifurcations, i.e. splitting of minimal invariant subsets into others within the attractor. Several examples are treated, including a nonautonomous pitchfork bifurcation.

Based on the paper; Bifurcation and continuous transition of attractors in autonomous and nonautonomous systems, Inter. J. Bifurcation & Chaos. 15 (2005), 743-762.

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Bifurcation analysis of aircraft on the ground

Bernd Krauskopf (Bristol)

Joint work with James Rankin, Phanikrishna Thota and Mark Lowenberg in collaboration with Airbus.

Modern passenger aircraft need to taxi from/to the runway as fast as possible, but in a safe and comfortable manner. The talk discusses how bifurcation therory can contribute to the understanding of stable aircraft taxi operations. Specifically, we consider the onset of shimmy oscillations during straight travel and the stability of high-speed turning.

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Invariant cones and bifurcation for nonsmooth systems

Tassilo Küpper (Köln)

Invariant manifolds, in particular center manifolds, play an essential part in establishing bifurcation results for (smooth) dynamical systems. In a recent paper the concept of invariant manifolds has been extended to piecewise-linear systems. In this lecture we will present a further extensions to more general piecewise smooth systems. In addition, bifurcation results will be established

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Continuation of cycle-to-cycle connections in 3D-ODEs

Yuri A. Kuznetsov (Utrecht)

Joint work with E. Doedel, B. Kooi and G.A.K. van Voorn.

It will be shown how easily one can locate and continue cycle-to-cycle connecting orbits in 3-dimensional autonomous ODEs. In our approach the projection boundary conditions near the cycles are formulated using eigenfunctions of the associated adjoint variational equations, thus avoiding computation of the monodromy matrices. The equations for the eigenfunctions are included in the defining boundary-value problem, allowing a straightforward implementation in AUTO, in which only the standard features of the software are used. Homotopy methods to find the connecting orbits will be discussed in general and illustrated with an example from population dynamics. In this food-chain model, a transverse homoclinic structure associated to a saddle cycle exists and can disappear via homoclininc tangencies, which we also compute.

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The discretized fold, transcritical and pitchfork bifurcations: conjugacy results

Lajos Lóczi (Budapest)

Let us consider a family of one-dimensional ODEs $\dot{x} = f(x, \alpha)$, where $\alpha \in \mathbb{R}$ denotes the bifurcation parameter. By constructing appropriate conjugacies in the vicinity of fold, transcritical and pitchfork bifurcation points, we compare the one-parameter family of time-h-maps of the ODEs and their stepsize-h discretizations of order p—the exact and discretized dynamics are shown to be topologically equivalent. We also estimate the distance between the conjugacy maps and the identity map in terms of h, α and p. In the case of transcritical and pitchfork bifurcations, further, in the $\alpha \leq 0$ case of the fold bifurcation containing the two branches of fixed points, we give optimal $\mathcal{O}(h^p)$ conjugacy estimates. However, the fixed point free $\alpha > 0$ case of the fold bifurcation seems harder to cope with: the problem of optimal conjugacy estimate is still open here.

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The Newton-Picard method for fluid flow problems

Kurt Lust (Groningen)

The Newton-Picard method is a time simulation based method for numerical bifurcation analysis of large scale systems. Since it can be used with many existing time simulation codes, it is a perfect extension to an engineer's simulation-based toolkit. The present method has been derived for large ODE systems such as those resulting from a space discretisation of a PDE. Many low-speed fluid flow problems are described by the Navier-Stokes equations for incompressible flow. These equations are an index 2 differential-algebraic equation when written down using the velocity and pressure variables. We will show that by making a good choice of our state parameters, we could use an existing simulation code without any modification to either the simulation code or the core of our Newton-Picard implementation.

In this talk, we will consider two applications: 2D flow behind a cylinder and axisymmetric flow in a combustor. For the first example we used a Navier-Stokes code developed in Groningen based on a low-order physics-preserving discretisation. The combustor case was done in collaboration with ETH Zürich using one of their spectral element codes.

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AUTO-07p and Computing Unstable Manifolds of Periodic Orbits in the Restricted 3-Body Problem

Bart Oldeman (Montreal)

Joint work with Eusebius Doedel.

AUTO-07p is an updated version of the well-known continuation package AUTO. It succeeds both AUTO97 and AUTO2000. An overview and explanation of its new capabilities is given, including the parallelization techniques that are used, scripting in Python, and visualization.

We show how AUTO can be used to compute unstable manifolds of periodic orbits in the restricted three body problem and present several examples, including homoclinic and heteroclinic connections.

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Resetting behavior of bursting in secretory pituitary cells

Hinke Osinga (Bristol)

Joint work with Andrew LeBeau, Arthur Sherman and Julie Stern, NIH Maryland.

We study a class of models for pituitary cells for which the spikes of the active phase are transient oscillations generated by unstable limit cycles emanating from a subcritical Hopf bifurcation around a stable steady state. We discuss the distinct properties of the response to attempted resets from the silent phase to the active phase. In particular, while resetting is difficult and succeeds only in limited windows of the silent phase, paradoxically, it can dramatically exceed the native active phase duration.

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Discretizing Dynamical Systems with a Codimension two Singularity

Joseph Paez Chavez (Bielefeld)

We consider one-step discretization methods of order $p \geq 1$ applied to a system of ordinary differential equations of the form

$$\dot{x}(t) = f(x(t), \beta, \alpha),$$

$(x(t), \beta, \alpha) \in \mathbb{R}^N \times \mathbb{R} \times \mathbb{R}$, where it is assumed that the system undergoes a codimension two bifurcation at $(x, \beta, \alpha) = (0, 0, 0)$. In particular we consider Bogdanov-Takens and fold-Hopf singularities. We show that a Bogdanov-Takens point is turned into a $1 : 1$ Resonance by Runge-Kutta methods. On the other hand a fold-Hopf point is $O(h^p)$-shifted and turned into a fold-Neimark-Sacker point by a general one-step discretization method. Next we analyze how a general one-step method discretizes the emanating curve of Hopf points in both cases. Our results are illustrated by a numerical example.

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Dynamics of neuronal coding

Khashayar Pakdaman (Paris)

Biological neurons operate as devices that transform incoming signals into trains of stereotyped brief electrical pulses referred to as spikes. The process of spike generation is inherently non linear and neurons are subject to various sources of internal and external fluctuations broadly referred to as noise. This talk will go over some aspects of neuronal coding that have been analyzed thanks to the technics coming from deterministic and stochastic dynamical system theory.

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A classical approach to nonautonomous continuation and bifurcation

Christian Pötzsche (München)

We investigate local continuation and bifurcation properties for nonautonomous equations. Extending a well-established autonomous theory, due to our quite arbitrary time dependence, equilibria or periodic solutions might not exist and are replaced by bounded complete solutions.

Following this leitmotiv, hyperbolicity is formulated via the dichotomy spectrum and yields a robust framework for local continuation arguments using the (surjective) implicit function theorem. Exponential dichotomies in variation also provide the necessary Fredholm theory in order to employ a Lyapunov-Schmidt-reduction to deduce nonautonomous versions of the classical fold, transcritical and pitchfork bifurcation patterns.

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Numerical bifurcation analysis of a PDE model for multisection semiconductor lasers

Mindaugas Radziunas (Berlin)

The traveling wave model consisting of a hyperbolic system of linear one space dimensional first order PDE's nonlinearly coupled with a system of ODE's is used to model the dynamics of multi-section semiconductor lasers. A low dimensional system of ODE's based on the projection of the original system into the subspace spanned only on a few instantaneously changing spectral elements allow a precise approximation of the original model. The numerical bifurcation analysis of the reduced system provides a better understanding of multi-section laser dynamics. This understanding have helped us to develop a new laser design concept for applications in optical communication systems.

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Bifurcation theory for nonautonomous differential equations

Martin Rasmussen (Augsburg)

Although, bifurcation theory for equations with autonomous and periodic time dependence is a major object of research in the study of dynamical systems since decades, the notion of a nonautonomous bifurcation is not yet established. In this talk, two approaches to overcome this deficit are presented in the context of nonautonomous differential equations. Based on special notions of attractivity and repulsivity, nonautonomous bifurcation phenomena are studied. We obtain generalizations of the well-known one-dimensional transcritical and pitchfork bifurcation.

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A Lin's method approach to finding and continuing heteroclinic connections involving periodic orbits

Thorsten Rieß (Ilmenau)

We present a numerical method for finding and continuing heteroclinic connections of vector fields that involve periodic orbits. Specifically, we concentrate on the case of a codimension-d heteroclinic connection from a saddle equilibrium to a saddle periodic orbit, denoted EtoP connection for short. By employing a Lin's method approach we construct a boundary value problem that has as its solution two orbit segments, one from the equilibrium to a suitable section Σ and the other from Σ to the periodic orbit. The difference between their two end points in Σ can be chosen in a d-dimensional subspace, and this gives rise to d well-defined test functions that are called the Lin gaps. A connecting orbit can be found in a systematic way by closing the Lin gaps one-by-one in d consecutive continuation runs. Indeed, any common zero of the Lin gaps corresponds to an EtoP connection, which can then be continued in system parameters.

To demonstrate the performance of our method, we consider a three-dimensional model vector field for the dynamics near a saddle-node Hopf bifurcation with global reinjection and show that our method allows us to complete a complicated bifurcation diagram involving codimension-one EtoP connections.

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Parameter estimation for hyperbolic partial differential equations, modelling virus population dynamics

Dirk Roose (Leuven)

Joint work with Tatyana Luzyanina and Gennady Bocharov.

The functioning of the immune system involves tightly regulated proliferation, differentiation and death processes of heterogeneous cell populations. The proliferation of e.g. T-lymphocyte cells, labelled with a fluorescence (CFSE) marker, defining the division age of the cell, can be studied experimentally by flow cytometry analysis.

We present a mathematical model for the dynamics of the cell distribution with respect to the intensity level of the CFSE-marker, that takes the form of a first order hyperbolic partial differential equation in one 'space' variable, i.e., the intensity level of the marker. We indicate that the numerical solution of the model can be computed accurately by the Lax-Wendroff method.

The parameters of the model are the rate functions of cell division, death, label decay and the label dilution factor, that need to be estimated from the flow cytometry data.. We present a computational approach to the identification of the model parameters with focus on the cell birth rate as a function of the marker intensity. To solve the inverse problem numerically, we parameterize the birth rate function and apply a maximum likelihood approach. Ill-posedness of the inverse problem is indicated by multiple minima. To treat the ill-posed problem, we apply Tikhonov regularization procedure. The solution of the regularized parameter estimation problem is consistent with the data set with an accuracy within the noise level in the measurements.

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A new development platform for computational bifurcation analysis

Frank Schilder (Surrey)

Joint work with Harry Dankowicz.

Computational bifurcation analysis relies on the combined application of covering algorithms to continue families of solutions to some problem; on means to locate special points along such families, for example, bifurcation points where families of the same or different type merge; and on methods for branch-switching, where families emerging from such special points are computed. The final result of such computations is a network of families connected at bifurcation points or ending at pre-defined computational boundaries, commonly referred to as a bifurcation diagram.

An advanced computational bifurcation analysis typically involves many more tasks than the ones sketched above, for example, the introduction of additional solution constraints. This algorithmic complexity often translates directly into complex implementations of bifurcation codes. It further imposes substantial costs for development and maintenance in terms of precious research time and, hence, forms a substantial obstacle for progress.

The development we present in this talk aims at breaking this 'curse of complexity.' We try to identify key tasks that are unique to all bifurcation computations and propose an implementation that performs these tasks in a standardised and fully transparent way. Whenever possible, our approach is to split our code into fully independent sub-toolboxes to simplify usage, maintenance, and replacement of portions of code. Standardisation, transparency, and modularity also enable independent development of bifurcation toolboxes on top of the core implementation. This presentation intends to stimulate a discussion of further steps to take, or problem classes to include, so as to make this development as useful as possible.

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Spatially complex localisation of an elastic conducting rod in a uniform magnetic field

David Sinden (London)

We consider the equilibrium equations of a conducting rod in a uniform magnetic field, motivated by the problem of electrodynamic tethers. The governing equations are found to be noncanonical reversible Hamiltonian equations. Homoclinic to trivial periodic solution are computed using shooting methods and continued using projection boundary conditions. A codimension-two Hamiltonian Hopf-

Hopf bifurcation is identified which acts as an organising centre for primary and multimodal homoclinics.

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A global numerical method for finding isolated solution branches with applications to a conducting elastic rod in a uniform magnetic field

András Árpád Sipos (Budapest)

Joint work with G.H.M. van der Heijden.

We carry out a numerical bifurcation study of a system of equations describing the equilibrium shapes of a conducting elastic rod moving in a uniform magnetic field as a model for an electrodynamic space tether. The equations and a novel set of boundary conditions were formulated in [1] where it was also shown that the initially straight rod becomes unstable at certain critical values of the magnetic field (or electric current). Post-buckling solutions were found to be helical. Numerical continuation software such as AUTO is able to compute branches of these bifurcating solutions. However, these may not be the only solutions of interest and this continuation approach would not find any solutions without a connection to the trivial path of straight rods by one or more bifurcations. To locate such solutions requires a global approach based on a scanning in the space of the free variables associated with the boundary conditions at one end of the rod. In fairly high dimensional problems as ours this approach carries a high computational cost. Here we apply the iteration-free Parallel Hybrid Algorithm (PHA) described in [2] to do the scanning in order to feed AUTO with a set of disconnected starting solutions. We find new solutions corresponding to non-helical shapes of the rod and present a bifurcation analysis of these solutions.

[1] G.H.M. van der Heijden, J. Valverde: Instability of a whirling conducting rod in the presence of a magnetic field. ENOC-2005, Eindhoven, Netherlands, 7-12. August, 2005.

[2] Zs. Gaspar, G. Domokos, I. Szebernyi: A parallel algorithm for the global computation of elastic bar structures. 4. (1997), Comput. Assist. Mech. Eng. Sci., pp. 55-68.

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Resonances in a piecewise-smooth system

Robert Szalai (Bristol)

Joint work with Hinke M. Osinga.

Resonance in smooth dynamical systems is a typical phenomenon leading to Arnol'd tongues in a two-parameter bifurcation diagram. Numerical observations show that for piecewise-smooth systems the resonance tongues look like strings of connected sausages. We explain this for the normal form of the Poincare map derived at a grazing-sliding bifurcation, which is similar to the smooth Neimark-Sacker bifurcation. We focus on the case where the dynamics on the one-dimensional global attractor of the normal form is phase locked. The associated periodic orbits bifurcate in border-collision bifurcations that lead to more complicated dynamics. Since in most models of physical systems non-smoothness is a simplifying approximation, we also relate our results to regularised systems.

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Lyapunov-Schmidt — a discrete revision

André Vanderbauwhede (Gent)

In this talk we present a slightly modified version of the Lyapunov-Schmidt reduction method applied to the bifurcation of periodic points from fixed points in discrete systems. There will be some emphasis on the relation between the Lyapunov-Schmidt approach and the center manifold and normal form reductions. The method will be illustrated with the problem of subharmonic bifurcations in autonomous ordinary differential equations and the associated appearance of Arnol'd tongues in parameter space.

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Snakes, ladders and isolas: Localised patterns in the Swift-Hohenberg equation

Thomas Wagenknecht (Leeds)

Stable localised roll structures have been observed in many physical problems and model equations, notably in the 1D Swift Hohenberg equation. Reflection-symmetric localised rolls are often found to lie on two snaking solution branches, so that the spatial width of the localised rolls increases when moving along each branch. In addition, recent numerical results indicate that the two branches are connected by infinitely many ladder branches of asymmetric localised rolls.

In this talk we use a dynamical systems approach to analyze these phenomena. We show that both snaking of symmetric pulses and the ladder structure of asymmetric states can be predicted completely from the bifurcation structure of fronts that connect the trivial state to rolls.

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Convergence properties of collocation methods for solving the adjoint equations

Virginie De Witte (Gent)

Joint work with Willy Govaerts.

The seminal paper "Collocation at Gaussian points" by Carl de Boor and Blair Swartz (SIAM J. Numer. Anal. 10(1973)582-606) laid the theoretical foundations for the use of collocation methods in the approximation of the solution of boundary value problems by piecewise polynomials. In particular, they showed that the use of the Gaussian points (zeroes of the Legendre polynomials) as collocation points leads to a faster convergence than, e.g., equidistributed mesh points.

Since then, approximation by piecewise polynomials and collocation at Gauss points has been the standard in dynamical systems software such as AUTO, CONTENT and MATCONT.

Adjoint problems to boundary value problems have attracted the attention for decades since they are closely related to sensitivity issues. They can be solved in a variety of ways. Recently, in the case of periodic orbits the solution to the adjoint problem was obtained as a byproduct of the computation of the periodic orbit itself (Yu.A. Kuznetsov, W. Govaerts, E.J. Doedel and A. Dhooge, Numerical periodic normalization for codim 1 bifurcations of limit cycles, SIAM J. Numer. Anal. 43 (2005) 1407-1435).

Since then, this idea has been applied in the computation of the critical normal form coefficients of bifurcation points, in the computation of the phase response curve and in the computation of test functions for bifurcations of homoclinic orbits. However, a study of the convergence properties of this method is still absent.

We discuss these convergence properties and find that they are again strongly dependent on the choice of the Gaussian points as collocation points. We also provide extensive numerical tests.

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Numerical bifurcation of Hamiltonian relative periodic orbits

Claudia Wulff (Surrey)

Relative periodic orbits (RPOs) are ubiquitous in symmetric Hamiltonian systems and occur for example in celestial mechanics, molecular dynamics and rigid body motion. RPOs are solutions which are periodic orbits of the symmetry-reduced system. In this talk we analyze certain symmetry-changing bifurcations of Hamiltonian relative periodic orbits and show how they can be detected and computed numerically. These are relative period-doubling bifurcations and relative Lyapounov centre bifurcations along branches of RPOs. We apply our methods to the families of rotating choreographies which bifurcate from the famous Figure Eight solution of the three body problem.