QE "Optimization", WS 2017/18

Problem Set No. 10

Submit your solutions by **20.11.2017**.

The problems will be discussed in the tutorials.

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1. [**2 points**] Let $f\colon U \to \mathbb{R}$ be a C^2-function defined on an open set $U \subset \mathbb{R}^2$. At the point (x_0, y_0) the function $f(x, y)$ has

$$f_x = f_y = 0, \quad f_{xx} = f_{yy} = 0 \quad \text{and} \quad f_{xy} = f_{yx} = 1.$$

What sort of critical point does f have at (x_0, y_0)?

2. [**4 Points**] Find the critical points and classify them when

$$f(x, y) := x^3 + y^3 - 3xy.$$

Hint: 1 saddle point, 1 local (but not global) minimum.

3. [**9 Points**] Consider the function f defined for all $x, y \in \mathbb{R}$ by

$$f(x, y) = xe^{-x}(y^2 - 4y).$$

(i) Find all stationary points of f and classify them by using the 2nd derivative test.

(ii) Show that f has neither a global maximum nor a global minimum.

(iii) Let

$$S := \{(x, y) \mid 0 \leq x \leq 5, 0 \leq y \leq 4\}.$$

Explain why you can be sure that f has global maximum and minimum points in S.

(iv) Find all global extreme points of f in S. Explain why you can be sure that these are the global extrema.

Hint: (i) 1 local min and 2 saddle points;

Hint: (ii) Study $f(x, 1)$ as $x \to \infty$ and $f(-1, y)$ as $y \to \infty$.

Hint: (iv) min is $-4/e$ achieved at $(1, 2)$, max is 0 at all $(x, 0)$ and $(x, 4)$ with $x \in [0, 5]$ and at all $(0, y)$ with $y \in [0, 4]$.

4. [**6 Points**] A firm produces a single commodity and gets p for each unit sold. The cost of producing x units is $ax + bx^2$ and the tax per unit is t. Assume that the parameters are positive with $p > a + t$. The firm wants to maximize profits.

(i) Find the optimal production x^* and the optimal profit π^*.

(ii) Using the envelope theorem, prove that $\partial\pi^*/\partial p = x^*$ and give an economic interpretation.