Dr. D. Frettlöh 13.12.2022

Formal Logic — Exercise Sheet 9

Exercise 33: (Herbrand universes and Herbrand expansions)

Let $F = \neg\exists x\, Q(x) \wedge \neg\big(\exists y\, \forall x\, P(x,y) \wedge \forall x\, \neg Q(x)\big)$.

(a) List all terms of the Herbrand universe $H(F)$ of F.
(b) Establish THE normal form (TNF) of F and list seven terms of the Herbrand universe of this TNF.
(c) List four terms of the Herbrand expansion $E(F)$ of the TNF of F.

Exercise 34: (THE normal form and resolution)
Establish THE normal form of

$$F = \forall x\, \forall y\, \Big(\neg\big(Q(f(a),x) \wedge \neg R(f(a))\big) \wedge \big(P(x) \Rightarrow Q(y,a)\big)\Big) \wedge \forall x\, \neg\big(P(a) \Rightarrow R(x)\big)$$

and use the resolution calculus in order to show that F is unsatisfiable.

Exercise 35: (I can't get no satisfaction)
Establish THE normal form of

$$F = \forall x\, \big(P(x) \vee Q(x)\big) \wedge \forall x\big(\neg Q(x) \vee Q(f(x))\big)$$

and apply the resolution calculus. Is F satisfiable?

Exercise 36: (Barber paradox)
In a remote small town there are two strict rules for barbers:

1. Each barber shaves all those who do not shave themselves.

2. No barber shaves someone who shaves himself.

Show that there is no barber in this town, by translating 1. and 2. and "there is a barber" into formulas F, G and H in first-order logic and use the resolution calculus in order to show that $F \wedge G \wedge H$ is unsatisfiable. *(Hint: use one predicate for "x is barber" and another one for "x shaves y".)*

Send your solutions until Tue 20.12.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.
Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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