From PET to SPLIT

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Abstract: Various forms of the polynomial ergodic theorem (PET) which attracted substantial attention in ergodic theory study the limits of expressions having the form $1/N\sum_{n=1}^{N}T^{q_1(n)}f_1\cdots T^{q_\ell(n)}f_\ell$ where T is a weakly mixing measure preserving transformation, f_i's are bounded measurable functions and q_i's are polynomials taking on integer values on the integers. Motivated partially by these results we obtain a central limit theorem for expressions of the form
$1/\sqrt{N}\sum_{n=1}^{N}(X_1(q_1(n))X_2(q_2(n))\cdots X_\ell(q_\ell(n))-a_1a_2\cdots a_\ell)$
(sum-product limit theorem–SPLIT) where X_i's are fast α-mixing bounded stationary processes, $a_j=EX_j(0)$ and q_i's are positive functions taking on integer values on integers with some growth conditions which are satisfied, for instance, when q_i's are polynomials of growing degrees. This result can be applied to the case when $X_i(n)=T^nf_i$ where T is a mixing subshift of finite type, a hyperbolic diffeomorphism or an expanding transformation taken with a Gibbs invariant measure, as well, as to the case when $X_i(n)=f_i(\xi_n)$ where ξ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure.

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