ICM 1998, Berlin, Aug. 18–27

Abstracts of Short Communications and Posters

Section:

2. Algebra

1991 MS Classification: 12, 13, 18

Börger, Reinhard and Kemper, Ralf, FernUniversität Hagen, Germany

Infinitary Linear Combinations in Cotorsion Modules

Let R be an integral domain such that the R-topology (with the collection of all non-zero ideals as a basis of zero-neighbourhoods) is complete and first-countable. If M is a cotorsion R-module in the sense of Matlis (i. e. $\mathrm{Hom}(K, M) = 0$ and $\mathrm{Ext}^1(K, M) = 0$ for $K := \mathrm{Quot}(R)$), we can define infinitary linear combinations $\sum_{n\in\mathbb{N}} \alpha_n x_n$ in M for all $\alpha_n \in R$, $x_n \in M$ with $\alpha_n \to 0$ in the R-topology extending the module structure such that $\sum_{n\in\mathbb{N}} \alpha_n(\sum_{m\in\mathbb{N}} \beta_{n,m} x_m) = \sum_{m\in\mathbb{N}}(\sum_{n\in\mathbb{N}} \alpha_n \beta_{nm}) x_m$ holds whenever the left-hand-side is defined. Moreover, this extension is unique, and each homomorphism between cotorsion modules preserves these infinitary linear combinations. Conversely, each module admitting infinitary linear combinations as above is cotorsion.