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Abstracts of Plenary and Invited Lectures

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0. Plenary Lectures

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Geometric Physics

Over the past two decades there has been growing interaction between theoretical physics and pure mathematics. Many of these connections have led to profound improvement in our understanding of physics as well as of mathematics. The aim of my talk is to give a non-technical review of some of these developments connected with string theory.

The central phenomenon in many of these links involves the notion of *duality*, which in some sense is a non-linear infinite dimensional generalization of Fourier transform. It suggests that two physical systems with completely different looking properties are nevertheless isomorphic if one takes into account "quantum geometry" on both sides. For many questions one side is simple (quantum geometry is isomorphic to classical one) and the other is hard (quantum geometry deforms the classical one). The equivalence of the systems gives rise to a rich set of mathematical identities. One of the best known examples of duality is known as "mirror symmetry" which relates topologically distinct pairs of Calabi-Yau manifolds and has applications in enumerative geometry. Other examples involve highly non-trivial "S-dualities" which among other things have found application to the study of smooth four manifold invariants. There have also been applications to questions of quantum gravity. In particular certain properties (the area of the horizon) of black hole solutions to Einstein equations have been related to growth of the cohomology of moduli space of certain minimal submanifold in a Calabi-Yau threefold.

A central theme in applications of dualities is a physical interpretation of singularities of manifolds. The most well known example is the $A-D-E$ singularities of $K3$ manifold which lead to $A-D-E$ gauge symmetry in the physical setup. The geometry of vanishing cycles is a key ingredient in the physical interpretation of singularities. More generally, singularities of manifolds encode universality classes of quantum field theories. This leads not only to a deeper understanding of the singularities of manifolds but can also be used to "geometrically engineer" new quantum field theories for physics. Through this method many non-trivial properties of quantum field theories get related to classical symmetries of geometry. An example of this application (in the context of Montonen-Olive duality) is a simple geometric interpretation of the modular properties of the generating function of the Euler class of moduli space of instantons on four manifolds.