ICM 1998, Berlin, Aug. 18–27

Abstracts of Plenary and Invited Lectures

Section:

8. Analysis

1991 MS Classification: 35 49
Müller, Stefan, MPI for Mathematics in the Sciences, Leipzig, Germany
Unexpected solutions of first and second order partial differential equations

The talk will present a general approach to construct Lipschitz solutions of $Du \in K$, where $u : \Omega \subset \mathbf{R}^n \to \mathbf{R}^m$ and where K is a given set of $m \times n$ matrices, based on (an extension of) Gromov's method of convex integration. One application concerns variational problems that arise in models of microstructure in solid-solid phase transitions. Another application is the systematic construction of singular solutions of elliptic systems. Specifically one has the following result.

Theorem. *There exists a 2×2 second order strongly elliptic system*

$$\operatorname{div} a(Du) = 0$$

that admits a nontrivial Lipschitz solution $u : \mathbf{R}^2 \to \mathbf{R}^2$ with compact support.

One can also contruct such 2×2 systems that arise as the Euler-Lagrange equations of a variational integral. By scaling one easily checks that there exist solutions whose singular set has almost full measure.
The talk is based on joint work with Vladimir Šverák that was supported by the Humboldt foundation through the Max Planck research prize.