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Abstracts of Plenary and Invited Lectures

Section:

8. Analysis

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Reflection Principle in Higher Dimensions

The reflection principle will be discussed with respect to the following
Conjecture. *Let $D, D' \subset\subset \mathbb{C}^n\,(n>1)$ be bounded domains with real-analytic boundaries and let let $f: D \to D'$ be a biholomorphic (or, more generally, proper holomorphic) mapping. Then f extends holomorphically to an open neighborhood of $\overline{D}$.*
For $n=1$ this is the classical Schwartz theorem and the reflection principle is the main tool for proving it. If the boundary ∂D of $D \subset \mathbb{C}$ is locally defined by an equation $\rho(z,\overline{z})=0$ and $\rho(z,\overline{w})$ is the complexification of $\rho(z,\overline{z})$, then the equation $\rho(z,\overline{w})=0$ can be locally solved as $z=h(w)$, where h is antiholomorphic, $h(w)=w$ for $w \in \partial D$ and $h \circ h(w) = w$ for all w. This defines an antiholomorphic reflection about ∂D. If h' is a reflection about $\partial D'$, then the analytic continuation of f across ∂D is given by $\tilde{f}(w) = h' \circ f \circ h(w)$.
For $n>1$ an analogous reflection about hypersurface ∂D cannot be immediately defined because in order to solve $\rho(z,\overline{w})=0$ for $z=(z_1,...,z_n)$ one needs at least $n-1$ more equations.
There are two main types of the reflection principle for $n>1$: analytic and geometric.
The idea of *the analytic reflection principle* is to differentiate f near ∂D in complex tangent directions in order to obtain remaining equations. This approach strongly depends on a preliminary study of boundary behaviour of f and was realized only in special situations (for example, for strictly pseudoconvex domains).
The geometric reflection principle is based on the technique of Segre varieties. For a given point w its Segre variety is a complex hypersurface $Q_w = \{z: \ \rho(z,\overline{w})=0\}$. This method was first introduced by S. Webster. Later significant contributions were done by K. Diederich and J. E. Fornaess. It was the main tool of the recent proof of the conjecture above for $n=2$, obtained by K. Diederich and S. Pinchuk.
The talk will be focused on the geometric reflection principle.