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12. Probability and Statistics

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Applications of Intentionally Biased Bootstrap Methods

For centuries the sample mean has been recognised as an estimator of the population mean — or in contemporary notation, $\bar{X} = \int x \, d\widehat{F}(x)$ is an estimator of $\mu = \int x \, dF(x)$, where $\widehat{F}$ denotes the empirical distribution function computed using a sample drawn from a distribution F. The idea that the sample median is an estimator of the population median is implicit in work of Galton about 120 years ago. Thus, the notion that a parameter may be regarded as a functional of a distribution function, and estimated by the same functional of the standard empirical distribution, is a rather old one, even though it was perhaps only recognised as a general principle relatively recently.
Efron's (1979) classic paper on the bootstrap vaulted statistical science forward from these simple ideas. Efron saw that when substituting the true F by an estimator $\widehat{F}$, the notion of a 'parameter' could be interpreted much more widely than ever before. It could include endpoints of confidence intervals or critical points of hypothesis tests, as well as error rates of discrimination rules. It could encompass tuning parameters in a wide variety of estimation procedures (even the nominal levels of intervals or tests can be regarded as tuning parameters), and much more.
Another key ingredient of the methods discussed by Efron (1979) was recognition that in cases where the functional of $\widehat{F}$ could not be computed directly, it could be approximated to arbitrary accuracy by Monte Carlo methods. The combination of these two ideas — the substitution or 'plug in $\widehat{F}$' rule, and the notion that Monte Carlo methods can be used to surmount computational obstacles — has been little short of revolutionary.
In 'standard' settings, where the appropriate way of applying the bootstrap is relatively clear, the uniform bootstrap offers an unambiguous approach to inference. Therein lies part of its attraction — there are no tuning parameters to be selected, for example. However, the lack of ambiguity can also be a drawback. In particular, the rigidity of the conventional bootstrap algorithm makes it relatively difficult to modify uniform-bootstrap methods so as to include constraints on the parameter space.
The weighted bootstrap, where resampling weights are not required to be identical, offers a way around this difficulty. It provides an opportunity for 'biasing' bootstrap estimators so as to fulfill constraints. Moreover, we may interpret the notion of a 'constraint' in a very broad sense, like that

of a 'parameter'. Nevertheless, an unambiguous approach to choosing the weights is required. Biased-bootstrap methods provide a solution to that problem.
The biased bootstrap requires two inputs from the experimenter: a measure of the distance of the weighted-bootstrap distribution from the uniform-bootstrap distribution, and the constraints. The first is generic to a wide range of problems, and will be discussed from that viewpoint in the lecture. The second is problem-specific, and will be introduced through a range of examples. A general form of the biased bootstrap consists of choosing the weights so as to minimise distance subject to the constraints being satisfied. The method encompasses Owen's (1988) technique of empirical likelihood, as well as related techniques such as Chen's (1997) approach to constrained density estimation, and a novel approach to bootstrapping the generalised method of moments model suggested by Imbens, Johnson and Spady (1998).

References,

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