A criterion for a central simple algebra to be split

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Abstract. In these notes we give a criterion for a central simple algebra A to be split in terms of the essential dimension of the algebraic group $\mathbf{SL}_1(A)$. This criterion also provides an example of an algebraic group G with $\mathrm{ed}(G) = n$ which does not possess any non-trivial cohomological invariant.

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1. Introduction

Let A be a central simple algebra over a field k. For every field extension K/k we will denote by A_K the algebra $A \otimes_k K$. We consider the algebraic group $\mathbf{SL}_1(A)$, defined over k, of elements of reduced norm equal to 1. This group is defined by the exact sequence of algebraic groups

$$1 \longrightarrow \mathbf{SL}_1(A) \longrightarrow \mathbf{GL}_1(A) \xrightarrow{\mathrm{Nrd}} \mathbb{G}_m \longrightarrow 1 \qquad (1)$$

where $\mathbf{GL}_1(A)$ stands for the algebraic group defined by $\mathbf{GL}_1(A)(K) = A_K^\times$ and $\mathrm{Nrd} : A_K^\times \to K^\times$ is the reduced norm homomorphism.

Applying Galois cohomology to the exact sequence (1), one finds, for every field extension K/k, the exact sequence of pointed sets

$$\mathbf{GL}_1(A)(K) \xrightarrow{\mathrm{Nrd}} \mathbb{G}_m(K) \xrightarrow{\partial} H^1(K, \mathbf{SL}_1(A)) \longrightarrow H^1(K, \mathbf{GL}_1(A)) \ .$$

Since $H^1(K, \mathbf{GL}_1(A)) = 1$ for all K/k (see [4], pp. 392-393), one obtains a surjection

$$\partial : \ \mathbb{G}_m(K) \twoheadrightarrow H^1(K, \mathbf{SL}_1(A)) \qquad (2)$$

and, much more precisely, an isomorphism

$$H^1(K, \mathbf{SL}_1(A)) \simeq K^\times / \mathrm{Nrd}(A_K^\times) \qquad (3)$$

which is functorial in K/k (see [4], Corollary 28.2, p.385 for details).

We will give a criterion for A to be split in terms of the triviality of the functor $K \mapsto H^1(K, \mathbf{SL}_1(A))$.

2. Essential dimension

Let k be a field. We denote by $\mathfrak{C}_k$ the category of field extensions of k, i.e. the category whose objects are field extensions K over k and whose morphisms are field homomorphisms which fix k. We will consider *covariant* functors $\mathbf{F} : \mathfrak{C}_k \to \mathbf{Sets}$ from $\mathfrak{C}_k$ to the category of sets. For such a functor $\mathbf{F}$ and for a field extension K/k, we will write $\mathbf{F}(K)$ instead of $\mathbf{F}(K/k)$. We shall say that a morphism $\mathbf{F} \longrightarrow \mathbf{G}$ between functors is a **surjection** if, for any field extension K/k, the corresponding map $\mathbf{F}(K) \longrightarrow \mathbf{G}(K)$ is a surjection of sets.

Definition 2.1. *Let* $\mathbf{F} : \mathfrak{C}_k \to \mathbf{Sets}$ *be a covariant functor,* K/k *a field extension and* $a \in \mathbf{F}(K)$*. For* $n \in \mathbb{N}$*, we say that the* **essential dimension of** a **is** $\leq n$ *(and we write* $\mathrm{ed}(a) \leq n$*) if there exists a subextension* E/k *of* K/k *such that:*

i) the transcendence degree of E/k *is* $\leq n$*,*

ii) the element a *is in the image of the map* $\mathbf{F}(E) \longrightarrow \mathbf{F}(K)$*.*

We say that $\mathrm{ed}(a) = n$ *if* $\mathrm{ed}(a) \leq n$ *and* $\mathrm{ed}(a) \not\leq n-1$*. The* **essential dimension of F** *is the supremum of* $\mathrm{ed}(a)$ *for all* $a \in \mathbf{F}(K)$ *and for all* K/k*. The essential dimension of* $\mathbf{F}$ *will be denoted by* $\mathrm{ed}_k(\mathbf{F})$*.*

Examples 2.2.

i) Consider the trivial functor $* : \mathfrak{C}_k \to \mathbf{Sets}$ which sends each K/k to a one-element set $*$. Clearly one has $\mathrm{ed}_k(*) = 0$.
ii) The functor $\mathbb{G}_m$, which assigns to each K the set $K^\times$, satisfies $\mathrm{ed}_k(\mathbb{G}_m) = 1$.
iii) Let $\mathbf{F} \twoheadrightarrow \mathbf{G}$ be a surjection. Then $\mathrm{ed}_k(\mathbf{G}) \leq \mathrm{ed}_k(\mathbf{F})$.

These facts are easy consequences of the definition. See [1] or [6] for example.

Definition 2.3. *For an algebraic group* G *defined over* k*, the essential dimension of the Galois cohomology functor* $K \mapsto H^1(K, G)$ *is denoted by* $\mathrm{ed}_k(G)$*.*

For an account on the notion of essential dimension of algebraic groups see for instance [1, 2, 6] or [7].

It follows from surjection (2) and from Examples 2.2 ii) and iii) above that $\mathrm{ed}_k(\mathbf{SL}_1(A)) \leq 1$. The question is to know when this number is equal to 0 or 1. If A is split, say $A \simeq M_n(k)$, then $\mathbf{SL}_1(A) \simeq \mathbf{SL}_n$ and the reduced norm homomorphism is clearly surjective. Hence the functor $K \mapsto K^\times/\mathrm{Nrd}(A_K^\times)$ is trivial and thus $\mathrm{ed}_k(\mathbf{SL}_1(A)) = 0$. We will see that the converse is true.

3. Statement of the result

Let t be an indeterminate and denote by $[t]$ the class of t in $k(t)^\times/\mathrm{Nrd}(A_{k(t)}^\times)$.

Theorem 3.1. *Let A be a central simple algebra over k. The following conditions are equivalent:*

1) *A is split,*
2) *the functor $K \mapsto K^\times/\mathrm{Nrd}(A_K^\times)$ is trivial,*
3) $\mathrm{ed}_k(\mathbf{SL}_1(A)) = 0$*,*
4) $\mathrm{ed}([t]) = 0$*.*

Proof. The implications 1) $\Rightarrow$ 2) $\Rightarrow$ 3) $\Rightarrow$ 4) are trivial and follow easily from the definitions. It remains to show that 4) $\Rightarrow$ 1). It will be a consequence of the following fact. □

Proposition 3.2. *Let A be a central simple algebra over k. Denote by* $\mathrm{ind}(A)$ *its index and consider the group homomorphism*

$$\theta_A : \ (A \otimes_k k(t))^\times \xrightarrow{\mathrm{Nrd}} k(t)^\times \xrightarrow{\deg} \mathbb{Z}$$

where deg *stands for the degree in t. Then* $\mathrm{im}(\theta_A) = \mathrm{ind}(A)\,\mathbb{Z}$*.*

To prove this fact, we use the following lemma.

Lemma 3.3. *Let K be a commutative field and A and B two simple central algebras over K. If A and B are Brauer equivalent, then* $\mathrm{Nrd}(A^\times) = \mathrm{Nrd}(B^\times)$*.*

Proof. Let D be a central division algebra over K such that $A \simeq M_r(D)$ and $B \simeq M_s(D)$. We show that $\mathrm{Nrd}(A^\times) = \mathrm{Nrd}(D^\times)$. We have that $A^\times \simeq \mathbf{GL}_r(D)$. Now for invertible matrices over a division algebra there is the notion of Dieudonné determinant $\det_D : \mathbf{GL}_r(D) \to D^\times/[D^\times, D^\times]$ (see [3], Definition 3, p. 135). One has the following commutative diagram

$$\begin{array}{ccc} \mathbf{GL}_r(D) & \xrightarrow{\mathrm{Nrd}} & K^\times \\ {\scriptstyle \det_D}\big\downarrow & \nearrow{\scriptstyle \mathrm{Nrd}} & \\ D^\times/[D^\times, D^\times] & & \end{array}$$

which shows that $\mathrm{Nrd}(A^\times) = \mathrm{Nrd}(D^\times)$ since $\det_D$ is surjective (see [3], Theorem 1, p. 146). Similarly $\mathrm{Nrd}(B^\times) = \mathrm{Nrd}(D^\times)$ which finishes the proof. □

Remark 3.4. It follows from the above lemma and the functor isomorphism (3) that, if A and B are two simple central algebras over k which are Brauer equivalent, then the functors $K \to H^1(K, \mathbf{SL}_1(A))$ and $K \to H^1(K, \mathbf{SL}_1(B))$ are isomorphic.

Proof of Proposition 3.2. By Lemma 3.3, we can suppose that A is a division algebra. In this case, the index of A and the degree of A are equal.

Consider now the element $1 \otimes t \in (A \otimes_k k(t))^{\times}$. We have that $\mathrm{Nrd}(1 \otimes t) = t^{\mathrm{ind}(A)}$ and thus $\mathrm{ind}(A) \in \mathrm{im}(\theta_A)$. It follows that $\mathrm{ind}(A)\,\mathbb{Z} \subseteq \mathrm{im}(\theta_A)$.
To show the converse inclusion take $a \in (A \otimes_k k(t))^{\times}$ and write it $a = \frac{b}{c}$ where $c \in k[t]$ and $b \in A \otimes_k k[t]$. Since $c \in k[t]$ we have that $\mathrm{Nrd}(c) = c^{\mathrm{ind}(A)}$ and thus it suffices to show that $\mathrm{ind}(A)$ divides the degree of $\mathrm{Nrd}(b)$. We then write

$$b = b_0 \otimes 1 + b_1 \otimes t + \cdots + b_n \otimes t^n$$

where $b_i \in A$ and $b_n \neq 0$. An easy computation shows that

$$\mathrm{Nrd}(b) = \mathrm{Nrd}(b_n)\, t^{n \cdot \mathrm{ind}(A)} + \cdots$$

Since A is a division algebra $\mathrm{Nrd}(b_n) \neq 0$ whence the result. □

Proposition 3.2 enables us to establish the implication 4) ⇒ 1) of Theorem 3.1.

Proof of Theorem 3.1. Suppose that $\mathrm{ed}([t]) = 0$. This means that there exist an element $\lambda \in k^{\times}$ and an element $x \in (A \otimes_k k(t))^{\times}$ such that $t = \lambda\,\mathrm{Nrd}(x)$. But this implies that $1 \in \mathrm{im}(\theta_A)$. By the preceeding Proposition, this means that $\mathrm{ind}(A) = 1$, saying that A is split. □

Actually Theorem 3.1 can be sharpened in order to give a more precise result:

Corollary 3.5. *Let A be a central simple algebra over k. Let $r \in \mathbb{N}$, then*

i) one has $\mathrm{ed}([t^r]) = 0$ if and only if $\mathrm{ind}(A)$ divides r;

ii) one has $\mathrm{ind}(A) = r$ if and only if $\mathrm{ed}([t^r]) = 0$ and $\mathrm{ed}([t^m]) \neq 0$ for all $m \neq r$ with $m \,|\, r$.

Proof. By Lemma 3.3 one can suppose A to be a division algebra.
i) Suppose that $\mathrm{ind}(A)$ divides r, say $r = \mathrm{ind}(A)m$. One has $t^r = \mathrm{Nrd}(1 \otimes t^m)$ saying that $\mathrm{ed}([t^r]) = 0$. Conversely, if $\mathrm{ed}([t^r]) = 0$ then there exists $u \in k$ and $x \in (A \otimes_k k(t))^{\times}$ such that $t^r = u\mathrm{Nrd}(x)$. This shows that $r \in \mathrm{im}(\theta_A)$. By Proposition 3.2, it follows that $\mathrm{ind}(A)$ divides r.
ii) It follows easily from i). □

4. A slight generalization

For two algebraic groups G and H very little is known about the behaviour of $\mathrm{ed}(G \times H)$ (see [1, 2] or [7] for partial results). Even when $G = H$ the computation of the essential dimension is not well understood. The preceeding discussion and the interpretation of the essential dimension of $\mathbf{SL}_1(A)$ in terms of the algebra A allows to give a precise description in this case. Our aim is to prove the following result for a product of copies of $\mathbf{SL}_1(A)$.

THEOREM 4.1. *Let A be a central simple algebra defined over a field k and let $\mathbf{SL}_1(A) \times \cdots \times \mathbf{SL}_1(A)$ the product of n copies of $\mathbf{SL}_1(A)$. Then*

$$\mathrm{ed}_k(\mathbf{SL}_1(A) \times \cdots \times \mathbf{SL}_1(A)) = n\, \mathrm{ed}_k(\mathbf{SL}_1(A)).$$

Proof. It is enough to prove that if A is a non-split algebra then we have $\mathrm{ed}_k(\mathbf{SL}_1(A) \times \cdots \times \mathbf{SL}_1(A)) = n$. Notice that, since there is no non-split central simple algebra over a finite field, we may suppose that the ground field k is infinite. For simplicity we will denote the functor $H^1(\_, \mathbf{SL}_1(A) \times \cdots \times \mathbf{SL}_1(A))$, with n copies of $\mathbf{SL}_1(A)$, by $\mathbf{F}_n$.

Let $t_1, \ldots, t_n$ be algebraically independent variables over k. In the sequel we will denote by K_n the field $k(t_1, \ldots, t_n)$. Consider the element $a = (\bar{t}_1, \ldots, \bar{t}_n) \in \mathbf{F}_n(K_n)$ where $\bar{t}_i$ denotes the class of t_i in the quotient $K_n^\times / \mathrm{Nrd}(A_{K_n}^\times)$. We will show, by induction on n, that $\mathrm{ed}(a) = n$.

Suppose that $n \geq 2$ and that there is a subextension $L/k \subset K_n/k$ and an element $b \in \mathbf{F}_n(L)$ such that b maps to a under the map

$$\mathbf{F}_n(L) \to \mathbf{F}_n(k(t_1, \ldots, t_n)).$$

Suppose moreover that $\mathrm{trdeg}(L : k) < n$. We write $b = (\bar{b}_1, \ldots, \bar{b}_n)$ for some elements $b_i \in L^\times$. Saying that b maps to a means that there exist elements $x_i \in (A \otimes_k K_n)^\times$ such that

$$\begin{cases} t_1 &= b_1 \,\mathrm{Nrd}(x_1) \\ t_2 &= b_2 \,\mathrm{Nrd}(x_2) \\ & \vdots \\ t_n &= b_n \,\mathrm{Nrd}(x_n) \end{cases}$$

Recall that for a field extension K/k a k-valuation on K is a valuation v on K which is trivial on k, that is $v(x) = 0$ for all $x \in k$. We denote by $\mathcal{O}_v = \{x \in K \mid v(x) \geq 0\}$ the ring of the valuation, by $\mathfrak{m}_v = \{x \in K \mid v(x) > 0\}$ its unique maximal ideal and by κ_v the residue field, that is the quotient $\mathcal{O}_v/\mathfrak{m}_v$. Since v is trivial on k it follows that $\mathcal{O}_v$ is a k-algebra and that κ_v is again a field extension of k. It is well known that

$$\mathrm{trdeg}(\kappa_v : k) \leq \mathrm{trdeg}(K : k)$$

and that the equality holds if and only if v is trivial on K.

We consider now on K_n the $(t_n - \lambda)$-valuation where $\lambda \in k$. We denote it by v and its restriction to L by v'. Since the field k is infinite, we can find a λ such that the b_i's, the x_i's and their inverses y_i are unramified at v. That means that $b_i \in \mathcal{O}_v$ and that $x_i, y_i \in A \otimes_k \mathcal{O}_v$. Thus we may specialise the above equations at $t_n = \lambda$ and find

$$\begin{cases} t_1 &= b_1(\lambda)\,\mathrm{Nrd}(x_1(\lambda)) \\ t_2 &= b_2(\lambda)\,\mathrm{Nrd}(x_2(\lambda)) \\ & \vdots \\ \lambda &= b_n(\lambda)\,\mathrm{Nrd}(x_n(\lambda)) \end{cases}$$

where $b_i(\lambda)$ denotes the image of b_i in the residue field $\kappa_{v'}$ and $x_i(\lambda)$ denote the image of x_i in $(A \otimes_k \kappa_v)^\times = (A \otimes_k K_{n-1})^\times$.
Hence $b' = (\bar{b}_1, \ldots, \bar{b}_{n-1}) \in \mathbf{F}_{n-1}(\kappa_{v'})$ maps to $(\bar{t}_1, \ldots, \bar{t}_{n-1}) \in \mathbf{F}_{n-1}(K_{n-1})$ under the induced map $\mathbf{F}_{n-1}(\kappa_{v'}) \to \mathbf{F}_{n-1}(\kappa_v)$. Since by induction hypothesis $\mathrm{ed}((\bar{t}_1, \ldots, \bar{t}_{n-1})) = n-1$ and since $\mathrm{trdeg}(L : k) < n$ it follows that the valuation v' has to be trivial over L. Consequently $b_n(\lambda) = b_n \in L$ and the equation $\lambda = b_n(\lambda)\,\mathrm{Nrd}(x_n(\lambda))$ actually shows that $b_n \in K_{n-1}$. Coming back to the equation $t_n = b_n\,\mathrm{Nrd}(x_n)$ this contradicts Theorem 3.1 for the central simple algebra $A' = A \otimes_k K_{n-1}$. □

5. A side remark

Let $\mathbf{F} : \mathfrak{C}_k \to \mathbf{Sets}$ be a covariant functor and let M be any torsion Γ_k-module. A **degree d cohomological invariant for F with values in M** is, by definition, a morphism of functors $\eta : \mathbf{F} \to H^d(\_, M)$. An invariant η is said to be non-trivial if, for every field extension K/k, there exists a field extension L/K and an element $a \in \mathbf{F}(L)$ such that $\eta_L(a) \neq 0 \in H^d(L, M)$. It is shown in [1, 6, 7] that if η is a non-trivial degree d cohomological invariant then $\mathrm{ed}_k(\mathbf{F}) \geq d$.
Now, since the algebra A splits over some field extension K/k, it follows that for any L/K the cohomology set $H^1(L, \mathbf{SL}_1(A))$ is reduced to one element. Thus one sees that $H^1(\_, \mathbf{SL}_1(A))$ cannot have any non-trivial cohomological invariant of degree ≥ 1. Hence, when A is non-split, $\mathbf{SL}_1(A) \times \cdots \times \mathbf{SL}_1(A)$ gives an easy example of a group without any non-trivial cohomological invariant which has essential dimension n.

6. Description of $H^1(k(t), \mathbf{SL}_1(A))$

For completeness we give a detailed study of $H^1(k(t), \mathbf{SL}_1(A))$ inspired by [8]. Let X denote the set of irreducible monic polynomials in $k[t]$.

Proposition 6.1. *Let A be a central simple algebra over k and $x \in X$. Consider the group homomorphism*

$$\eta_x : \ (A \otimes_k k(t))^\times \xrightarrow{\mathrm{Nrd}} k(t)^\times \xrightarrow{v_x} \mathbb{Z}$$

where v_x denotes the x-adic valuation on $k(t)$. Then $\mathrm{im}(\eta_x) = \mathrm{ind}(A)\mathbb{Z}$.

Proof. As in Lemma 3.3 we can suppose that A is a division algebra. One has $\mathrm{Nrd}(1 \otimes x) = x^{\mathrm{ind}(A)}$, thus $\mathrm{ind}(A)\mathbb{Z} \subseteq \mathrm{im}(\eta_x)$. Conversely, let $a \in (A \otimes_k k(t))^\times$ and write it $a = \frac{b}{c}$ where $c \in k[t]$ and $b \in A \otimes_k k[t]$. Since $\mathrm{Nrd}(c) = c^{\mathrm{ind}(A)}$ it suffices to prove that $\mathrm{ind}(A) \mid v_x(\mathrm{Nrd}(b))$. To do this, write

$$b = \sum_{r \le i \le s} b_i \otimes x^i f_i$$

where $\deg(f_i) < \deg(x)$ and $b_r \neq 0$. An easy computation shows then that

$$\mathrm{Nrd}(b) = \mathrm{Nrd}(b_r) x^{r \cdot \mathrm{ind}(A)} f_r^{\mathrm{ind}(A)} + x^{r \cdot \mathrm{ind}(A)+1} g$$

where $g \in k[t]$. Since $\mathrm{Nrd}(b_r) \neq 0$ this shows that $v_x(\mathrm{Nrd}(b)) = r \cdot \mathrm{ind}(A)$. Hence $\mathrm{im}(\eta_x) \subseteq \mathrm{ind}(A)\mathbb{Z}$. This concludes the proof. □

Definition 6.2. *For each $x \in X$, let*

$$\partial_x : H^1(k(t), \mathbf{SL}_1(A)) \to \mathbb{Z}/\mathrm{ind}(A)\mathbb{Z}$$

be the group homomorphism induced by the valuation v_x.

Theorem 6.3. *Let A be a central simple algebra over k. Then there is a split exact sequence*

$$1 \to H^1(k, \mathbf{SL}_1(A)) \to H^1(k(t), \mathbf{SL}_1(A)) \to \bigoplus_{x \in X} \mathbb{Z}/\mathrm{ind}(A)\mathbb{Z} \to 0$$

where the first map is induced by $k \to k(t)$ and the second is $\oplus \partial_x$.

Proof. Though it is well known that the natural map

$$\iota : H^1(k, G) \to H^1(k(t), G)$$

is injective (see [5]), we show injectivity in this particular case. As above we may suppose that A is a division algebra. So let $\lambda \in k^\times$ and $x \in (A \otimes k(t))^\times$ such that $\lambda = \mathrm{Nrd}(x)$. Write x as $x = (1/p(t))y$, where $p(t)$ is a unitary polynomial and $y \in A \otimes k[t]$. One has

$$\mathrm{Nrd}(x) = (1/p(t)^{\deg(A \otimes k(t))})\mathrm{Nrd}(y),$$

and hence $\lambda p(t)^{\mathrm{ind}(A)} = \mathrm{Nrd}(y)$. Write $y = a_r \otimes t^r + \cdots + a_0 \otimes 1$, where $a_i \in A$ and $a_r \neq 0$. Then $\mathrm{Nrd}(y)$ is a polynomial of degree $\mathrm{ind}(A)r$ and its leading coefficient is $\mathrm{Nrd}(a_r)$. Comparing the leading terms in the equality $\lambda p(t)^{\mathrm{ind}(A)} = \mathrm{Nrd}(y)$, one gets $\lambda = \mathrm{Nrd}(a_r)$. This proves the injectivity.

The composition

$$k^\times/\mathrm{Nrd}(A) \to k(t)^\times/\mathrm{Nrd}(A^\times_{k(t)}) \to \bigoplus_{x\in X} \mathbb{Z}/\mathrm{ind}(A)\mathbb{Z}$$

is clearly zero. We show that $\ker(\oplus\delta_x) \subseteq \mathrm{im}(\iota)$. Let $f \in k(t)^\times$ and write it $f = \lambda \prod_{x\in X} x^{v_x(f)}$ with $\lambda \in k^\times$. Since $\partial_x(f) = 0$ one has that $\mathrm{ind}(A)$ divides $v_x(f)$ for all $x \in X$. Hence $\prod_{x\in X} x^{v_x(f)} \in \mathrm{Nrd}(A^\times_{k(t)})$. Thus $f \in \mathrm{im}(\iota)$.

To end the proof, we give a section to $\oplus\partial_x$. This will show surjectivity as well. We let $s : \bigoplus_{x\in X} \mathbb{Z}/\mathrm{ind}(A)\mathbb{Z} \longrightarrow H^1(k(t), \mathbf{SL}_1(A))$ defined by sending the element $([n_x])_{x\in X}$ to the class of $\prod_{x\in X} x^{n_x}$ in $k(t)^\times/\mathrm{Nrd}(A^\times_{k(t)})$. This clearly gives the desired section. □

Corollary 6.4. *The following conditions are equivalent:*

i) *the algebra A is split,*
ii) *the group $H^1(k(t), \mathbf{SL}_1(A))$ is trivial,*
iii) *the group $H^1(k(t), \mathbf{SL}_1(A))$ is finite.*

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