ON PROPERTY $D(2)$ AND COMMON SPLITTING FIELD OF TWO BIQUATERNION ALGEBRAS

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Abstract. Let F be a field of characteristic $\neq 2$. We say that F has property $D(2)$ if for any quadratic extension L/F and any two binary quadratic forms over F having a common nonzero value over L this value can be chosen in F. We show that if k is a field of characteristic $\neq 2$ having at least two distinct quadratic extensions, then for the field $k(x)$ property $D(2)$ does not hold. Using this we construct two biquaternion algebras over a field $K = k(x)((t))((u))$ such that their sum is a quaternion algebra, but they do not have a common biquadratic (i.e. a field of the kind $K(\sqrt{a}, \sqrt{b})$, where $a, b \in K^*$) splitting field.

Let F be a field of characteristic different from 2. By definition, F has property $D(2)$ if for any quadratic extension L/F and any two binary quadratic forms q_1, q_2 over F the existence of a common value of the forms q_{1L}, q_{2L} implies the existence of a common value of the forms q_{1L}, q_{2L}, which lies in F. Examples of fields of characteristic 0 not satisfying this property has been given in [5]. Later in [1] starting from such a field, it has been shown that the answers to the following questions are negative in general:

1) *Let (a_1, b_1) and (a_2, b_2) be quaternion algebras over a field K. Suppose $c \in K^*$ is such that $(a_1, b_1) \otimes (a_2, b_2)_{K(\sqrt{c})}$ is not a division algebra. Is it true that there exists $d \in K^*$ such that*

$$(a_1, b_1)_{K(\sqrt{c}, \sqrt{d})} = (a_2, b_2)_{K(\sqrt{c}, \sqrt{d})} = 0 \ ?$$

2) *Let φ be an 8-dimensional form from $I^2(K)$ whose Clifford algebra has index 4. Is it true that φ is a direct sum of two forms similar to 2-fold Pfister forms ?*

3) *Let φ be a 14- dimensional form from $I^3(K)$. Is it true that φ is similar to the difference of the pure parts of two 3-fold Pfister forms ?*

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By means of quite a different and very nonelementary technique the negative answers to the same questions have been given in [3].

The above questions stipulate our interest to property $D(2)$. As far as we know, examples of fields of positive characteristic not satisfying property $D(2)$ are not known. In this note independently of characteristic of the field we give simple counterexamples to property $D(2)$ such that the 2-cohomological dimension of the ground field F equals 2. Using these examples we show that the following question has the negative answer in general:

Suppose D_1 and D_2 are two biquaternion algebras over the field K such that $\mathrm{ind}(D_1+D_2)=2$. *Does there exist a common splitting field of D_1 and D_2 of the type $K(\sqrt{a},\sqrt{b})$, where $a,b\in K^*$?*

Our notation is standard. All the fields in the sequel are of characteristic different from 2. By form we always mean a quadratic form over a field. Slightly abusing notation we often identify a form over F with the corresponding class in the Witt group $W(F)$. By φ_{an} we denote the anisotropic part of the form φ. If L/F is a field extension, and φ is a form over F, then $D_L(\varphi)$ is the set of nonzero values of φ_L, and $G_L(\varphi)$ is the group of multipliers of φ_L, i.e. $G_L(\varphi)=\{a\in L^* : \varphi\simeq a\varphi\}$. By the n-fold Pfister form $\langle\langle a_1,\dots,a_n\rangle\rangle$ we mean the form $\langle 1,-a_1\rangle\otimes\ldots\otimes\langle 1,-a_n\rangle$. By (a,b) we denote the quaternion algebra with generators $1,i,j,k$ and relations $i^2=a,\ j^2=b,\ ij=-ji$. If k is a field, t is an indeterminate and $p\in k[t]$ is a monic irreducible polynomial, then $\partial_p : W(k(t))\to W(k[t]/p)$ is the second residue map, i.e. the group homomorphism determined by the following rule: for a squarefree polynomial $f\in k[t]$

$$\partial_p(\langle f\rangle)=\begin{cases} 0 \text{ if } p \text{ does not divide } f, \\ \langle \frac{f}{p}\rangle \text{ if } p \text{ divides } f \end{cases}.$$

The tensor product of central simple K-algebras is always considered over the field K.

We turn to the construction of the examples in question. Let k be a field, $a,d_1,d_2\in k^*$ such that $d_1^2-4a,\ d_2^2-4a,\ (d_1^2-4a)(d_2^2-4a)\notin k^{*2}$, x an indeterminate, $F=k(x)$, $L=F(\sqrt{x^2-4a})$. Set

$$q_1\simeq (x-d_1)\langle\langle d_1^2-4a\rangle\rangle\in W(F),$$

$$q_2\simeq (x-d_2)\langle\langle d_2^2-4a\rangle\rangle\in W(F).$$

Proposition 1. *The field F does not satisfy property $D(2)$, and the forms q_1,q_2 and the quadratic extension L/F provide a counterexample.*

Proof. It is trivial to check that

$$\begin{aligned} &q_1((x-d_1-\sqrt{x^2-4a})(x-d_1)^{-1},(x-d_1)^{-1}) \\ &=q_2((x-d_2-\sqrt{x^2-4a})(x-d_2)^{-1},(x-d_2)^{-1}) \\ &=2(x-\sqrt{x^2-4a})\in L. \end{aligned}$$

This shows that $D_L(q_1)\cap D_L(q_2)\neq\emptyset$.

Now let us prove that $D_L(q_1)\cap D_L(q_2)\cap F=\emptyset$. We will follow an idea in [6]. Let y_1,y_2 be indeterminates, $\widehat{k}$ a maximal odd degree extension of $k(y_1,y_2)$. Since d_1^2-4a and d_2^2-4a are distinct nontrivial elements in $k^*/{k^*}^2$, it is easy to see that $(d_1^2-4a,y_1)\otimes_{\widehat{k}}(d_2^2-4a,y_2)$ is a division algebra. Put

$$\varphi=\langle\langle d_1^2-4a,y_1\rangle\rangle-\langle\langle d_2^2-4a,y_2\rangle\rangle.$$

In particular, $\varphi\neq 0$ over any quadratic extension of $\widehat{k}$. Put $y=\sqrt{x^2-4a},\ x-y=2t$. Then $x+y=\frac{2a}{t}$, hence $x=t+\frac{a}{t}$. So we have $L=F(\sqrt{x^2-4a})=k(t)$, $F=k(t+\frac{a}{t})$. Suppose that $u\in D_{\widehat{k}(t)}(q_1)\cap D_{\widehat{k}(t)}(q_2)\cap\widehat{k}(x)$. Obviously, we may assume that $u\in\widehat{k}[x]$ and $u=u(x)$ is squarefree. Then

$$(x-d_1)u\in D_{\widehat{k}(t)}(\langle\langle d_1^2-4a\rangle\rangle)=G_{\widehat{k}(t)}(\langle\langle d_1^2-4a\rangle\rangle)=G_{\widehat{k}(t)}(q_1),$$

$$(x-d_2)u\in D_{\widehat{k}(t)}(\langle\langle d_2^2-4a\rangle\rangle)=G_{\widehat{k}(t)}(\langle\langle d_2^2-4a\rangle\rangle)=G_{\widehat{k}(t)}(q_2).$$

Therefore, we have

$$\begin{aligned}(q_1\langle\langle y_1\rangle\rangle-q_2\langle\langle y_2\rangle\rangle)_{\widehat{k}(t)}&=((x-d_1)uq_1\langle\langle y_1\rangle\rangle-(x-d_2)uq_2\langle\langle y_2\rangle\rangle)_{\widehat{k}(t)}\\&=u(\langle\langle d_1^2-4a,y_1\rangle\rangle-\langle\langle d_2^2-4a,y_2\rangle\rangle)_{\widehat{k}(t)}=u\varphi_{\widehat{k}(t)}.\end{aligned}\tag{1}$$

Similarly, since $4t=2(x-\sqrt{x^2-4a})\in D_L(q_1)\cap D_L(q_2)$, we have

$$(q_1\langle\langle y_1\rangle\rangle-q_2\langle\langle y_2\rangle\rangle)_{\widehat{k}(t)}=t\varphi_{\widehat{k}(t)}.\tag{2}$$

Combining (1) and (2) we conclude that

$$u\varphi_{\widehat{k}(t)}=t\varphi_{\widehat{k}(t)}\tag{3}$$

Substituting x for $t+\frac{a}{t}$ in the left part of (3) and comparing the residues at t of the both parts of this equality we see that the degree of the polynomial u in x is odd. Since the field $\widehat{k}$ has no proper extensions of odd degree, we conclude that there is $c\in\widehat{k}$ such that $x-c$ divides u. Notice that $x-c=\frac{p(t)}{t}$, where $p(t)=t^2-ct+a$. Comparing the residues at $p(t)$ of the both parts of the equality (3) we get a contradiction, since $\partial_p(t\varphi)=0$, and $\partial_p(u\varphi)=\varphi_{\widehat{k}(\sqrt{c^2-4a})}\neq 0$. The proposition is proved.

Corollary 2. *Let k be a field, $b_1,b_2\in k^*$. Then the following conditions are equivalent.*

1) *The elements $1,b_1,b_2\in k^*/{k^*}^2$ are pairwise distinct.*

2) *The field $k(x)$ has not property $D(2)$ and there exist $a\in k^*$, $s_1,s_2\in k(x)^*$ such that the extension $k(x,\sqrt{x^2-4a})/k(x)$ and the forms $s_1\langle\langle b_1\rangle\rangle$ and $s_2\langle\langle b_2\rangle\rangle$ provide a counterexample.*

Proof. 1) $\Rightarrow$ 2). By Proposition 1 it suffices to find $a,d_1,d_2\in k^*$ such that $b_1=d_1^2-4a$, $b_2=d_2^2-4a$. One can put, for instance, $d_1=\frac{b_1-b_2+1}{2}$, $d_2=\frac{b_2-b_1+1}{2}$, $a=\frac{1}{4}(d_1^2-b_1)$.

2) $\Rightarrow$ 1). If b_1 is a square, then $D_L(\langle\langle b_1\rangle\rangle) = L^*$ for any extension $L/k(x)$, hence for any $s_1, s_2 \in k(x)^*$ the forms $s_1\langle\langle b_1\rangle\rangle$ and $s_2\langle\langle b_2\rangle\rangle$ do not provide a counterexample to property $D(2)$. Assume that b_1b_2 is a square, i.e. $b_1 \equiv b_2 (\mod k^{*2})$. Let $L/k(x)$ be an arbitrary field extension. Assume that $c \in D_L(s_1\langle\langle b_1\rangle\rangle) \cap D_L(s_2\langle\langle b_2\rangle\rangle)$. Then $\langle\langle b_1, cs_1\rangle\rangle = \langle\langle b_1, cs_2\rangle\rangle = 0$, hence $\langle\langle b_1, s_1s_2\rangle\rangle = 0$, and so

$$s_1 \in D_L(s_1\langle\langle b_1\rangle\rangle) \cap D_L(s_2\langle\langle b_2\rangle\rangle) \cap k(x).$$

Remark. If k_0 is an algebraically closed field, and k is the rational function field over k_0, then $cd_2(k) = 1$ and $cd_2(k(x)) = 2$. By Corollary 2 the field $k(x)$ does not have property $D(2)$. On the other hand, if $I^2(K) = 0$, then the field K has property $D(2)$, since 1 is a value of any binary form over K.

It has been established in [1] that if the field F has not property $D(2)$, then the field $F_1 = F((t))$ has not property CS. Recall that property CS for the field F_1 means that for any quaternion algebras Q_1, Q_2 over F_1 and $c \in F_1^*$ such that $(Q_1 \otimes Q_2)_{F_1(\sqrt{c})}$ is not a division algebra there exists $d \in F_1^*$ such that $Q_{1\,F_1(\sqrt{c},\sqrt{d})} = 0$, $Q_{2\,F_1(\sqrt{c},\sqrt{d})} = 0$. In fact, if the binary forms $s_1\langle\langle b_1\rangle\rangle$, $s_2\langle\langle b_2\rangle\rangle$ and the quadratic extension $F(\sqrt{c})/F$ provide a counterexample to property $D(2)$ for the field F, then the quaternion algebras $Q_1 \simeq (b_1, s_1t)$, $Q_2 \simeq (b_2, s_2t)$ give a counterexample to property CS for the field $F((t))$.

Let now F be an arbitrary field, for which property CS does not hold. Assume that quaternion algebras Q_1, Q_2 and a quadratic extension $F(\sqrt{c})/F$ provide a counterexample. In particular, $(Q_1 \otimes Q_2)_{F(\sqrt{c})}$ is not a division algebra, which implies that $Q_1 \otimes Q_2 \simeq (c, d) \otimes (e, f)$ for some $d, e, f \in F^*$. Notice that $(e, f) \neq 0$, for otherwise $Q_1$and Q_2 would have a common quadratic subfield. Consider the biquaternion algebras $D_1 \simeq Q_1 \otimes (c, u)$ and $D_2 \simeq Q_2 \otimes (c, du)$ over the field $F((u))$.

Proposition 3. 1) $\mathrm{ind}(D_1 \otimes D_2) = 2$.

2) *The algebras D_1 and D_2 do not have a common biquadratic splitting extension. In other words, for any $p, q \in F((u))^*$ either $D_{1\,F((u))(\sqrt{p},\sqrt{q})} \neq 0$, or $D_{2\,F((u))(\sqrt{p},\sqrt{q})} \neq 0$.*

Proof.

1) $D_1 + D_2 = Q_1 + Q_2 + (c, u) + (c, du) = Q_1 + Q_2 + (c, d) = (e, f)$, which proves the first part of the proposition.

2) Assume the contrary, i.e. that $D_{1\,F((u))(\sqrt{p},\sqrt{q})} = 0$, $D_{2\,F((u))(\sqrt{p},\sqrt{q})} = 0$. Since $F((u))^*/F((u))^{*2} = F^*/F^{*2} \oplus \mathbb{Z}/2\mathbb{Z}$, we may assume that $p \in F^*$. If $q \in F^*$, it is easy to see that $F(\sqrt{c}) \subset F(\sqrt{p}, \sqrt{q})$, i.e. $F(\sqrt{p}, \sqrt{q}) = F(\sqrt{c}, \sqrt{c'})$ for some $c' \in F^*$. Hence

$$Q_{1\,F(\sqrt{c},\sqrt{c'})} = Q_{2\,F(\sqrt{c},\sqrt{c'})} = 0,$$

which is impossible, since the algebras Q_1, Q_2 and the extension $F(\sqrt{c})/F$ provide a counterexample to property CS. Therefore, we may suppose that $q = au$, where $a \in F^*$. Then, obviously,

$$(Q_1 + (c, a))_{F((u))(\sqrt{p},\sqrt{au})} = (Q_1 + (c, u))_{F((u))(\sqrt{p},\sqrt{q})} = 0,$$

$$(Q_2 + (c, ad))_{F((u))(\sqrt{p},\sqrt{au})} = (Q_2 + (c, du))_{F((u))(\sqrt{p},\sqrt{q})} = 0.$$

Hence,

$$(Q_1 + (c,a))_{F(\sqrt{p})} = 0,\ (Q_2 + (c,ad))_{F(\sqrt{p})} = 0,$$

which implies that

$$Q_{1F(\sqrt{c},\sqrt{p})} = Q_{2F(\sqrt{c},\sqrt{p})} = 0,$$

a contradiction to the fact that the algebras Q_1, Q_2 and the extension $F(\sqrt{c})/F$ provide a counterexample to property $D(2)$. The proposition is proved.

Remark. To give a concrete example of biquaternion algebras D_1 and D_2 satisfying the conditions of Proposition 3 we start from the binary forms $q_1 \simeq (x-d_1)\langle\langle d_1^2-4a\rangle\rangle$ and $q_2 \simeq (x-d_2)\langle\langle d_2^2-4a\rangle\rangle$ over the field $k(x)$ from Proposition 1. Then $Q_1 \simeq (d_1^2-4a,(x-d_1)t)$, $Q_2 \simeq (d_2^2-4a,(x-d_2)t)$. It is easy to check the following equality comparing the residues on its both sides:

$$Q_1+Q_2 = (x^2-4a, -(x-d_1)(x-d_2)((d_1+d_2)x-4a-d_1d_2)) +$$
$$((d_1^2-4a)(d_2^2-4a), ((d_1+d_2)x-4a-d_1d_2)(d_1+d_2)^{-1}t).$$

Tracing back the construction of the algebras D_1, D_2 from the algebras Q_1, Q_2, we get over the field $k(x)((t))((u))$

$$D_1 \simeq (d_1^2-4a,(x-d_1)t)\otimes(x^2-4a,u),$$

$$D_2 \simeq (d_2^2-4a,(x-d_2)t)\otimes(x^2-4a,-(x-d_1)(x-d_2)((d_1+d_2)x-4a-d_1d_2)u).$$

Notice that if $cd_2k=1$, $K=k(x)((t))((u))$, then $cd_2K=4$, so one can construct a counterexample with a field K of 2-cohomological dimension 4. On the other hand, there is no such a counterexample for a field of cohomological dimension 2. More precisely we have the following

Proposition 4. *Let K be a field such that $I^3(K)=0$. Let further D_1, D_2 be biquaternion algebras such that* $\mathrm{ind}(D_1+D_2)=2$. *Then there exist $p,q\in K^*$ such that*

$$D_{1K(\sqrt{p},\sqrt{q})} = D_{2K(\sqrt{p},\sqrt{q})} = 0.$$

Proof. By ([2], Lemma 14.2) we have $I^3(K(\sqrt{a}))=0$ for any $a\in K^*$. Let φ_1, φ_2 be Albert forms corresponding to D_1, D_2. Then by ([4], Ch.2, Th.14.4) we get

$$(\varphi_1-\varphi_2)_{K(\sqrt{b})} \in I^3(K(\sqrt{b}))=0$$

for any $b\in K^*$ such that $(D_1+D_2)_{K(\sqrt{b})}=0$. Hence $\varphi_1 \perp -\varphi_2 \simeq \langle\langle b\rangle\rangle\otimes\psi$, where $\dim\psi=6$. Let $\psi_1\in I^2(K)$ be such a form that $\psi=\psi_1+\tau$, where $\dim\tau=2$. Then we have

$$\varphi_1-\varphi_2 = \langle\langle b\rangle\rangle\otimes\psi_1 + \langle\langle b\rangle\rangle\otimes\tau = \langle\langle b\rangle\rangle\tau,$$

since $\langle\langle b\rangle\rangle\otimes\psi_1\in I^3(K)=0$. Therefore, $\dim(\varphi_1\perp-\varphi_2)_{an}\le 4$. This means that φ_1 and φ_2 have a common 4-dimensional subform, say

$$\varphi_1 \simeq \langle a,b,c,d,e_1,-abcde_1\rangle,$$

$$\varphi_2 \simeq \langle a, b, c, d, e_2, -abcde_2 \rangle.$$

Then, obviously,

$$\varphi_{1K(\sqrt{-ab},\sqrt{-cd})} = \varphi_{2K(\sqrt{-ab},\sqrt{-cd})} = 0,$$

which is equivalent to

$$D_{1K(\sqrt{-ab},\sqrt{-cd})} = D_{2K(\sqrt{-ab},\sqrt{-cd})} = 0,$$

so we can put $p = -ab$, $q = -cd$. The proposition is proved.

The following theorem [7] is the main motivation of the present note, and for the sake of completeness we give here its proof.

Theorem 5. *Let D_1 and D_2 be biquaternion algebras over a field k such that $\mathrm{ind}(D_1 + D_2) = 2$. Then there exists a field extension l/k of degree 4 such that $D_{1l} = D_{2l} = 0$.*

Proof. Let $D_1 + D_2 = Q = (a, b)$. Then by the dimension count $D_1 \otimes Q \simeq M_2(D_2)$. In particular, $Q \hookrightarrow M_2(D_2)$. It follows that there exist $I, J \in M_2(D_2)$ such that $I^2 = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, $J^2 = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix}$, and $IJ = -JI$.

Let $I = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. We may assume that $\alpha^2 \neq a$, and D_2 is a division algebra (in the opposite case the theorem is simple and left to the reader).

Lemma 6. *There is a matrix $S \in GL_2(D_2)$ such that $SIS^{-1} = \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix}$.*

Proof. Since $\alpha^2 \neq a$ and $I^2 = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, we have $\beta \neq 0$. Then for any $x \in D_2$

$$\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix} I \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -x & 1 \end{pmatrix} = \begin{pmatrix} \alpha - \beta x & * \\ * & * \end{pmatrix}.$$

Hence setting $x = \beta^{-1}\alpha$ we may assume that $\alpha = 0$. Since $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}^2 = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, we get $\beta\gamma = a$, $\beta\delta = 0$. Since D_2 is a division algebra, we conclude that $\delta = 0$, i.e. $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} 0 & a\gamma^{-1} \\ \gamma & 0 \end{pmatrix}$. Put $C = \begin{pmatrix} 1 & 0 \\ 0 & a\gamma^{-1} \end{pmatrix}$. Then

$$C \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} C^{-1} = \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix}.$$

The lemma is proved.

In view of the above lemma, conjugating by a suitable matrix we may assume that $I = \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix}$. Let $J = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$. Since

$$\begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix} \begin{pmatrix} x & y \\ z & t \end{pmatrix} = -\begin{pmatrix} x & y \\ z & t \end{pmatrix} \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix},$$

we get that $z = -ay$, $t = -x$. Therefore, $\begin{pmatrix} x & y \\ -ay & -x \end{pmatrix}^2 = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix}$, which means that $x^2 - ay^2 = b$ and $xy = yx$. Hence the subalgebra $l_1 = k[x, y]$ is a subfield of D_2. Moreover, $Q_{l_1} = 0$. If l is a maximal subfield of D_2 containing l_1, then $Q_l = (D_2)_l = 0$, which implies $(D_1)_l = 0$, and proves Theorem 5.

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