TRANSFER OF QUADRATIC FORMS AND OF QUATERNION ALGEBRAS OVER QUADRATIC FIELD EXTENSIONS

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A well-known theorem of Albert states that if a tensor product of two quaternion division algebras Q_1, Q_2 over a field F of characteristic different from 2 is not a division algebra, then there exists a quadratic extension L of F that embeds as a subfield in Q_1 and in Q_2; see [6, (16.29)]. The same property holds in characteristic 2, with the additional condition that L/F is separable: this was proved by Draxl [2], and several proofs have been proposed: see [3, Th. 98.19], and [7] for a list of earlier references.

Our purpose in this note is to extend the Albert–Draxl Theorem by substituting for the tensor product of two quaternion algebras the corestriction of a single quaternion algebra over a quadratic extension. Our main result is the following:

0.1. **Theorem.** *Let F be an arbitrary field and let K be a quadratic étale F-algebra.*[1] *For every quaternion K-algebra Q, the following conditions are equivalent:*

(i) *Q contains a quadratic F-algebra linearly disjoint from K;*
(ii) *Q contains a quadratic étale F-algebra linearly disjoint from K;*
(iii) $\operatorname{Cor}_{K/F} Q$ *is not a division algebra.*

Note that when $K = F \times F$ the quaternion K-algebra Q has the form $Q_1 \times Q_2$ for some quaternion F-algebras Q_1, Q_2, and $\operatorname{Cor}_{K/F} Q = Q_1 \otimes_F Q_2$. Thus, in this particular case Theorem 0.1 is equivalent to the Albert–Draxl Theorem. The more general case is needed for the proof of the main result in [1].

If the characteristic $\operatorname{char} F$ is different from 2, Theorem 0.1 is proved in [6, (16.28)]. The proof below is close to that in [6], but it does not require any restriction on the characteristic. The idea is to use a transfer of the norm form n_Q of Q to obtain an Albert form of $\operatorname{Cor}_{K/F} Q$, which allows us

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[1] This means that K is either a quadratic separable field extension of F, or $K \simeq F \times F$.

to substitute for (iii) the condition that the transfer of n_Q has Witt index at least 2. To complete the argument, we need to relate totally isotropic subspaces of the transfer to subforms of n_Q defined over F. This is slightly more delicate in characteristic 2. Therefore, we first discuss the transfer of quadratic forms in §1, and give the proof of Theorem 0.1 in §2. In the last section, we sketch an alternative proof of Theorem 0.1 based on a proof of the Albert–Draxl Theorem due to Knus [5]. This alternative proof relies on an explicit construction of an Albert form for the corestriction of a quaternion algebra.

Notations and Terminology. *Quaternion algebras* over an arbitrary field F are F-algebras obtained from an étale quadratic F-algebra E and an element $a \in F^\times$ by the following construction:

$$(E/F, a) = E \oplus Ez$$

with multiplication defined by the equations

$$z^2 = a \qquad \text{and} \qquad z\ell = \iota(\ell)z \quad \text{for } \ell \in L,$$

where ι is the nontrivial automorphism of L.

For quadratic and bilinear forms, we generally follow the notation and terminology of [3]. Thus, if $\varphi\colon V \to F$ is a quadratic form on a (finite-dimensional) vector space V over an arbitrary field F, we let $\mathfrak{b}_\varphi\colon V \times V \to F$ denote the polar form of φ, defined by

$$\mathfrak{b}_\varphi(x, y) = \varphi(x + y) - \varphi(x) - \varphi(y) \qquad \text{for } x,\, y \in V.$$

We set

$$\begin{aligned} \operatorname{rad} \mathfrak{b}_\varphi &= \{x \in V \mid \mathfrak{b}_\varphi(x, y) = 0 \text{ for all } y \in V\} \\ \operatorname{rad} \varphi &= \{x \in \operatorname{rad} \mathfrak{b}_\varphi \mid \varphi(x) = 0\} \end{aligned}$$

and observe that these sets are F-subspaces of V with $\operatorname{rad} \varphi \subseteq \operatorname{rad} \mathfrak{b}_\varphi$. Moreover, if $\operatorname{char} F \neq 2$ then $\varphi(x) = \frac{1}{2}\mathfrak{b}_\varphi(x, x)$ for all $x \in V$ and thus $\operatorname{rad} \varphi = \operatorname{rad} \mathfrak{b}_\varphi$. We call the quadratic form φ *nonsingular*[2] if $\operatorname{rad} \mathfrak{b}_\varphi = \{0\}$, *regular* if $\operatorname{rad} \varphi = \{0\}$ and *nondegenerate* if φ_K is regular for every field extension K/F or equivalently (by [3, Lemma 7.16]) if φ is regular and $\dim_F \operatorname{rad} \mathfrak{b}_\varphi \leq 1$. Thus, every nonsingular form is nondegenerate and every nondegenerate form is regular; moreover, all three conditions are equivalent in the case where $\operatorname{char} F \neq 2$.

The *Witt index* of a quadratic form φ on a vector space V is the dimension of the maximal totally isotropic subspaces of V, i.e., the maximal subspaces $U \subseteq V$ such that $\varphi(u) = 0$ for all $u \in U$; see [3, Prop. 8.11]. We write $\mathfrak{i}_0(\varphi)$ for the Witt index of φ.

We will need the following easy observation:

[2]Nonsingular quadratic forms are not defined in [3].

0.2. **Lemma.** *Let φ be a regular quadratic form on a vector space V. If φ is isotropic, the isotropic vectors span V.*

Proof. Let $V_0 \subseteq V$ be the subspace spanned by the isotropic vectors of V, and let $v \in V \setminus \{0\}$ be an isotropic vector. Since $\mathrm{rad}(\varphi) = \{0\}$ there exists $w \in V$ such that $\mathfrak{b}_\varphi(v, w) = 1$. If $x \in V$ is such that $\mathfrak{b}_\varphi(v, x) \neq 0$, then the vector $x - \varphi(x)\mathfrak{b}_\varphi(v, x)^{-1}v$ is isotropic, hence it belongs to V_0. It follows that $x \in V_0$ since $v \in V_0$. Thus, V_0 contains all the vectors that are not orthogonal to v. In particular, it contains w. If $x \in V$ is orthogonal to v, then $\mathfrak{b}_\varphi(v, x+w) = 1$, hence $x+w \in V_0$, and therefore $x \in V_0$ since $w \in V_0$. Thus, $V_0 = V$. □

1. Isotropic transfers

Let F be an arbitrary field and let K be a quadratic field extension of F. Fix a nonzero F-linear functional $s\colon K \to F$ such that $s(1) = 0$. For every quadratic form $\varphi\colon V \to K$ on a K-vector space, the transfer $s_*\varphi$ is the quadratic form on V (viewed as an F-vector space) defined by

$$s_*\varphi(x) = s\bigl(\varphi(x)\bigr) \qquad \text{for } x \in V.$$

If φ is nonsingular, then $s_*\varphi$ is nonsingular: see [3, Lemma 20.4]. For every quadratic form ψ over F, we let ψ_K denote the quadratic form over K obtained from ψ by extending scalars to K.

The following result is well-known in characteristic different from 2, but it appears to be new in characteristic 2.

1.1. **Theorem.** *Let φ be a nonsingular quadratic form over K. Then there exists a nondegenerate quadratic form ψ over F with* $\dim\psi = \mathfrak{i}_0(s_*\varphi)$ *such that ψ_K is a subform of φ.*

Proof. Substituting for φ its anisotropic part, we may assume φ is anisotropic. We may also assume $\mathfrak{i}_0(s_*\varphi) \geq 1$, for otherwise there is nothing to show. Pick an isotropic vector $u \in V$ for the form $s_*\varphi$; we thus have $\varphi(u) \in F$ and $\varphi(u) \neq 0$ since φ is anisotropic. If $\mathfrak{i}_0(s_*\varphi) = 1$, then we may choose $\psi = \varphi|_{Fv}$. For the rest of the proof, we assume $\mathfrak{i}_0(s_*\varphi) \geq 2$, and we argue by induction on $\mathfrak{i}_0(s_*\varphi)$.

Since φ is nonsingular, we may find $v \in V$ such that $\mathfrak{b}_\varphi(u, v) = 1$. Let $\lambda \in K$ be such that $s(\lambda) = 1$. We have $s_*\varphi(u) = 0$ and

$$\mathfrak{b}_{s_*\varphi}(u, \lambda v) = s\bigl(\mathfrak{b}_\varphi(u, \lambda v)\bigr) = s(\lambda) = 1,$$

hence the restriction of $s_*\varphi$ to the F-subspace $U \subseteq V$ spanned by u and λv is nonsingular and isotropic. Therefore, $s_*\varphi|_U$ is hyperbolic. Let $W = U^\perp \subseteq V$ be the orthogonal complement of U in V for the form $s_*\varphi$. Since $\mathfrak{i}_0(s_*\varphi) \geq 2$, the form $s_*\varphi|_W$ is isotropic. By Lemma 0.2 we may find an isotropic vector $w \in W$ such that $\mathfrak{b}_\varphi(u, w) \neq 0$. However, $\mathfrak{b}_{s_*\varphi}(u, w) = 0$

since $w \in W = U^{\perp}$; therefore $\mathfrak{b}_{\varphi}(u,w) \in F^{\times}$. Moreover $\varphi(w) \in F$ since w is isotropic for $s_*\varphi$. Therefore, the restriction of φ to the F-subspace of V spanned by u and w is a quadratic form ψ_1 over F.

Observe that u and w are K-linearly independent: if $w = \alpha u$ with $\alpha \in K$, then

$$\mathfrak{b}_{\varphi}(u,w) = \alpha\mathfrak{b}_{\varphi}(u,u) = 2\alpha\varphi(u).$$

Since $\mathfrak{b}_{\varphi}(u,w) \in F^{\times}$ and $\varphi(u) \in F$, it follows that $\alpha \in F$, which is impossible. Therefore, ψ_{1K} is a 2-dimensional subform of φ, and ψ_1 is nonsingular because φ is anisotropic. Since $s_*(\psi_{1K})$ is hyperbolic, for the orthogonal complement φ' of ψ_{1K} in φ we obtain $\mathfrak{i}_0(s_*\varphi') = \mathfrak{i}_0(s_*\varphi) - 2$. The theorem follows by induction. □

Remarks. 1. If $s_*\varphi$ is hyperbolic, then $\mathfrak{i}_0(s_*\varphi) = \dim\varphi$, hence Theorem 1.1 shows that $\varphi = \psi_K$ for some quadratic form ψ over F. This particular case of Theorem 1.1 is established in [3, Th. 34.9].

2. If $\operatorname{char} F = 2$ and $\mathfrak{i}_0(s_*\varphi)$ is odd, then the quadratic form ψ in Theorem 1.1 cannot be nonsingular, since nonsingular quadratic forms in characteristic 2 are even-dimensional; in particular, ψ_K is not an orthogonal direct summand of φ.

3. If the extension K/F is purely inseparable, then $\mathfrak{i}_0(s_*\varphi)$ is necessarily even. This follows because the K-subspace spanned by each isotropic vector for $s_*\varphi$ is a 2-dimensional F-subspace that is totally isotropic for $s_*\varphi$.

4. The analogue of Theorem 1.1 for symmetric bilinear forms is proved in [3, Prop. 34.1].

2. Proof of Theorem 0.1

Let F be an arbitrary field. As in [3], we write $I_q(F)$ for the Witt group of nonsingular quadratic forms of even dimension over F, and $I(F)$ for the ideal of even-dimensional forms in the Witt ring $W(F)$ of nondegenerate symmetric bilinear forms over F, and we let $I_q^n(F) = I^{n-1}(F)I_q(F)$ for $n \geq 2$. Let also $\mathrm{Br}_2(F)$ denote the 2-torsion subgroup of the Brauer group of F. Recall from [3, Th. 14.3] the group homomorphism

$$e_2\colon I_q^2(F) \to \mathrm{Br}_2(F)$$

defined by mapping the Witt class of a quadratic form φ to the Brauer class of its Clifford algebra.

2.1. **Lemma.** *Let K be a quadratic field extension of an arbitrary field F, and let $s\colon K \to F$ be a nonzero F-linear functional such that $s(1) = 0$. The* *following diagram is commutative:*

$$\begin{array}{ccc} I_q^2(K) & \xrightarrow{s_*} & I_q^2(F) \\ {\scriptstyle e_2}\big\downarrow & & \big\downarrow{\scriptstyle e_2} \\ \mathrm{Br}_2(K) & \xrightarrow{\mathrm{Cor}_{K/F}} & \mathrm{Br}_2(F). \end{array}$$

Following the definition in [4, Rem. 6.9.2], $\mathrm{Cor}_{K/F}\colon \mathrm{Br}_2(K) \to \mathrm{Br}_2(F)$ is the zero map if K is a purely inseparable extension of F.

Proof. We have $I_q^2(K) = I(F)I_q(K) + I(K)I_q(F)$ by [3, Lemma 34.16], hence $I_q^2(K)$ is generated by Witt classes of 2-fold Pfister forms that have a slot in F. Commutativity of the diagram follows by Frobenius reciprocity [3, Prop. 20.2], the computation of transfers of 1-fold Pfister forms in [3, Lemma 34.14] and [3, Cor. 34.19], and the projection formula in cohomology [4, Prop. 3.4.10]. □

Proof of Theorem 0.1. Since (ii) ⇒ (i) is clear, it suffices to prove (i) ⇒ (iii) and (iii) ⇒ (ii).

If (i) holds, then we may represent Q in the form $(LK/K, b)$ where L is a quadratic étale F-algebra linearly disjoint from K and $b \in K^\times$, or in the form $(M/K, b)$ where M is a quadratic étale K-algebra and $b \in F^\times$. In each case the projection formula [4, Prop. 3.4.10] shows that $\mathrm{Cor}_{K/F} Q$ is Brauer-equivalent to a quaternion algebra, hence (iii) holds.

Now, assume (iii) holds. If Q is split, then it contains an F-algebra isomorphic to $F \times F$, so (ii) holds. For the rest of the proof, we assume Q is a division algebra. Let n_Q be the norm form of Q, which is a 2-fold Pfister quadratic form in $I_q^2(K)$ such that $e_2(n_Q) = Q$ in $\mathrm{Br}(K)$. Since n_Q represents 1, the transfer $s_*(n_Q)$ is isotropic, hence Witt-equivalent to a 6-dimensional nonsingular quadratic form φ in $I_q^2(F)$. This form satisfies $e_2(\varphi) = \mathrm{Cor}_{K/F}(Q)$ in $\mathrm{Br}(F)$, hence φ is an Albert form of $\mathrm{Cor}_{K/F}(Q)$ as per the definition in [6, (16.3)]. In particular, since $\mathrm{Cor}_{K/F}(Q)$ is not a division algebra, φ is isotropic by [6, (16.5)], and therefore $\mathfrak{i}_0\big(s_*(n_Q)\big) \geq 2$. By Theorem 1.1 there exists a nonsingular quadratic form ψ over F with $\dim \psi = 2$ such that ψ_K is a subform of n_Q. Since Q is a division algebra, we have that ψ_K is anisotropic, hence ψ is similar to the norm form of a unique separable quadratic field extension L/F. The field L is linearly disjoint from K over F because ψ_K is anisotropic. On the other hand, ψ_{KL} is hyperbolic, hence KL splits the form n_Q, and it follows that there exists a K-algebra embedding of KL in Q. Therefore, (ii) holds. □

If K is a purely inseparable quadratic extension of F, all the statements of Theorem 0.1 hold for every quaternion algebra over K. To see this, recall from the definition of $\mathrm{Cor}_{K/F}$ in [4, Rem. 6.9.2] that the corestriction of every quaternion K-algebra is split. Moreover, if $Q = (M/K, b)$ with M a separable quadratic extension of K, then the separable closure of F in M is a separable quadratic extension of F contained in Q and linearly disjoint from K.

3. The Albert form of a corestriction

Let Q be a quaternion algebra over a separable quadratic field extension K of an arbitrary field F. By definition (see [6, (16.3)]), the Albert forms of $\operatorname{Cor}_{K/F} Q$ are the 6-dimensional nonsingular quadratic forms in $I_q^2(F)$ such that $e_2(\varphi) = \operatorname{Cor}_{K/F} Q$ in $\operatorname{Br}_2(F)$; they are all similar. As observed in the proof of Theorem 0.1, an Albert form of $\operatorname{Cor}_{K/F} Q$ may be obtained from the Witt class of the (8-dimensional) transfer $s_*(n_Q)$ of the norm form of Q for an arbitrary nonzero F-linear functional $s\colon K \to F$ such that $s(1) = 0$. In this section, we sketch a more explicit construction of an Albert form of $\operatorname{Cor}_{K/F} Q$, inspired by Knus's proof of the Albert–Draxl Theorem in [5], and we use it to give an alternative proof of Theorem 0.1.

We first recall the construction of the corestriction $\operatorname{Cor}_{K/F} Q$. Let γ be the nontrivial F-automorphism of K and let ${}^{\gamma}Q$ denote the conjugate quaternion algebra ${}^{\gamma}Q = \{{}^{\gamma}x \mid x \in Q\}$ with the operations

$$ {}^{\gamma}x + {}^{\gamma}y = {}^{\gamma}(x+y), \quad {}^{\gamma}x \cdot {}^{\gamma}y = {}^{\gamma}(xy), \quad \lambda \cdot {}^{\gamma}x = {}^{\gamma}(\gamma(\lambda)x) $$

for x, $y \in Q$ and $\lambda \in K$. The algebra ${}^{\gamma}Q \otimes_K Q$ carries a γ-semilinear automorphism s defined by

$$ s({}^{\gamma}x \otimes y) = {}^{\gamma}y \otimes x \qquad \text{for } x,\ y \in Q. $$

By definition, the corestriction (or norm) $\operatorname{Cor}_{K/F}(Q)$ is the F-algebra of fixed points (see [6, (3.12)]):

$$ \operatorname{Cor}_{K/F}(Q) = \bigl({}^{\gamma}Q \otimes_K Q\bigr)^s. $$

Let Trd and Nrd denote the reduced trace and the reduced norm on Q. Let also σ be the canonical (conjugation) involution on Q. Consider the following K-subspace of ${}^{\gamma}Q \otimes_K Q$:

$$ V = \{{}^{\gamma}x_1 \otimes 1 - 1 \otimes x_2 \mid x_1, x_2 \in Q \text{ and } \gamma\bigl(\operatorname{Trd}(x_1)\bigr) = \operatorname{Trd}(x_2)\}. $$

This K-vector space has dimension 6 and is preserved by s, and one can show that the F-space of s-invariant elements has the following description, where $T_{K/F}\colon K \to F$ is the trace form:

$$ V^s = \{{}^{\gamma}y \otimes 1 + 1 \otimes y \mid y \in Q \text{ and } T_{K/F}(\operatorname{Trd}(y)) = 0\}. $$

Now, pick an element $\kappa \in K^\times$ such that $\gamma(\kappa) = -\kappa$. (If $\operatorname{char} F = 2$ we may pick $\kappa = 1$.) The following formula defines a quadratic form $\varphi\colon V^s \to F$: for $y \in Q$ such that $T_{K/F}(\operatorname{Trd}(y)) = 0$, let

$$ \varphi({}^{\gamma}y \otimes 1 + 1 \otimes y) = \kappa \cdot \bigl(\gamma(\operatorname{Nrd}(y)) - \operatorname{Nrd}(y)\bigr). $$

Nonsingularity of the form φ is easily checked after scalar extension to an algebraic closure of F, and computation shows that the linear map

$$f\colon V^s \to M_2\big(\mathrm{Cor}_{K/F}(Q)\big) \quad \text{given by} \quad \xi \mapsto \begin{pmatrix} 0 & \kappa\cdot(\sigma\otimes\mathsf{id})(\xi) \\ \xi & 0 \end{pmatrix}$$

satisfies $f(\xi)^2 = \varphi(\xi)$ for all $\xi \in V^s$. Therefore, f induces an F-algebra homomorphism f_* defined on the Clifford algebra $C(V^s,\varphi)$. Dimension count shows that f_* is an isomorphism

$$f_*\colon C(V^s,\varphi) \xrightarrow{\sim} M_2(\mathrm{Cor}_{K/F} Q). \tag{3.1}$$

The restriction to the even Clifford algebra is an isomorphism $C_0(V^s,\varphi) \simeq (\mathrm{Cor}_{K/F} Q) \times (\mathrm{Cor}_{K/F} Q)$, hence the discriminant (or Arf invariant) of φ is trivial. This means $\varphi \in I_q^2(F)$, and (3.1) shows that $e_2(\varphi) = \mathrm{Cor}_{K/F} Q$ in $\mathrm{Br}_2(F)$, so φ is an Albert form of $\mathrm{Cor}_{K/F} Q$.

We use the Albert form φ to sketch an alternative proof of Theorem 0.1. Since all the conditions in Theorem 0.1 trivially hold if Q is split, we may assume Q is a division algebra. In particular, the base field F is infinite.

Suppose condition (i) of Theorem 0.1 holds. If $x \in Q$ generates a quadratic F-algebra disjoint from K, then $\mathrm{Trd}(x) \in F$ and $\mathrm{Nrd}(x) \in F$ (and $x \notin K$), hence ${}^\gamma(\kappa x) \otimes 1 + 1 \otimes (\kappa x) \in V^s$ is an isotropic vector of φ. Since φ is an Albert form of $\mathrm{Cor}_{K/F} Q$, it follows that $\mathrm{Cor}_{K/F} Q$ is not a division algebra. Therefore, (i) implies (iii).

For the converse, suppose (iii) holds, and let ${}^\gamma y \otimes 1 + 1 \otimes y \in V^s$ be an isotropic vector for φ. Then $\gamma(\mathrm{Nrd}(y)) - \mathrm{Nrd}(y) = 0$, hence $\mathrm{Nrd}(y) \in F$, and $T_{K/F}(\mathrm{Trd}(y)) = 0$. Assuming $y \in K$ quickly leads to a contradiction, and a density argument shows that we may find such an element y with $\mathrm{Trd}(y) \neq 0$. Then $\kappa y \in Q$ satisfies

$$\mathrm{Trd}(\kappa y) \in F, \qquad \mathrm{Trd}(\kappa y) \neq 0, \quad \text{and} \quad \mathrm{Nrd}(\kappa y) = \kappa^2 \,\mathrm{Nrd}(y) \in F.$$

Therefore, κy generates a quadratic étale F-subalgebra of Q linearly disjoint from K, proving that (ii) (hence also (i)) holds.

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