Arithmetic groups and Rigidity
Talk 1: Introduction

Arithmetic groups…

Combine

- algebra,
- number theory,
- geometry,
- geometric group theory.

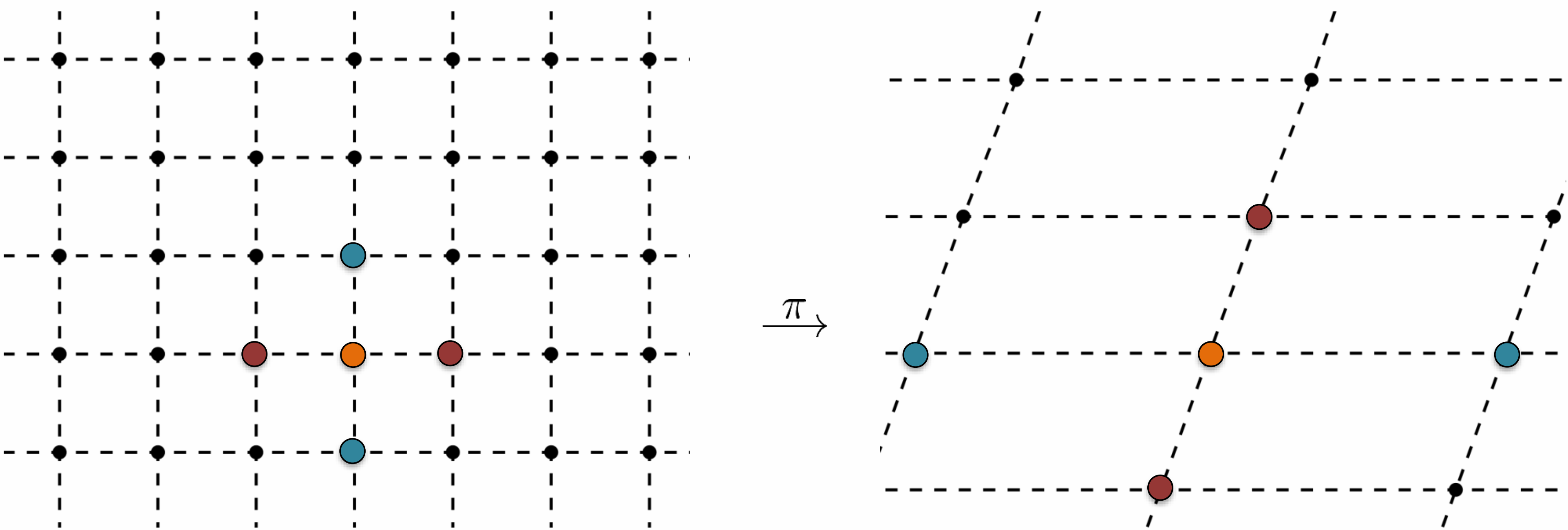
… **and** Margulis Super**rigidity**

- important consequences in algebra and geometry
- proof uses techniques from different areas
- influenced many other rigidity results

A rigidity theorem

Let $\Gamma_1 \leq \mathbb{R}^n$ and $\Gamma_2 \leq \mathbb{R}^m$ be discrete subgroups such that $\mathbb{R}^n/\Gamma_1$ and $\mathbb{R}^m/\Gamma_2$ are compact.

Then every group isomorphism $\pi : \Gamma_1 \to \Gamma_2$ extends to a continuous group isomorphism $\bar{\pi} : \mathbb{R}^n \to \mathbb{R}^m$.



$\Gamma_1 = \mathbb{Z}^2 \subset \mathbb{R}^2$

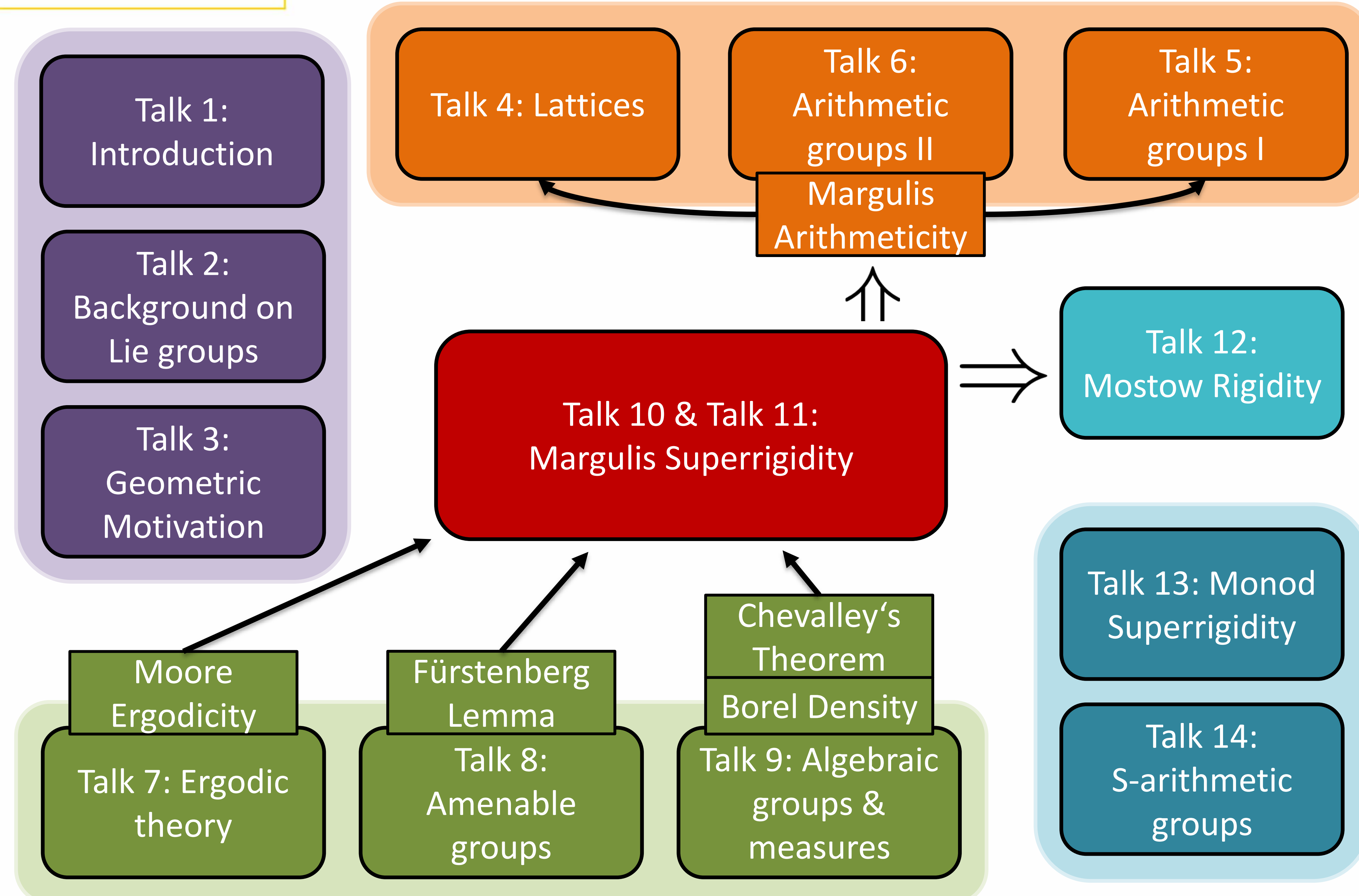
$\Gamma_2 = \left\langle (3,0),\, (\tfrac{1}{2}, \sqrt{2}) \right\rangle \subset \mathbb{R}^2$

Margulis‘ Superrigidity Theorem ('77)

Let G and H be connected algebraic $\mathbb{R}$-groups that satisfy the following conditions:

- G is semi-simple, has $\mathbb{R}$-rank greater than 1 and $G^0_{\mathbb{R}}$ has no compact factors.
- H is simple and $H_{\mathbb{R}}$ is not compact.

Let $\Gamma \subset G^0_{\mathbb{R}}$ be an irreducible lattice. Then every homomorphism $\pi : \Gamma \to H$ whose image is Zariski dense extends to a rational homomorphism $\bar{\pi} : G \to H$ which is defined over $\mathbb{R}$.


Talk 1: Introduction
Talk 2: Background on Lie groups
Talk 3: Geometric Motivation
Talk 4: Lattices
Talk 6: Arithmetic groups II
Talk 5: Arithmetic groups I
Margulis Arithmeticity
Talk 10 & Talk 11: Margulis Superrigidity
Talk 12: Mostow Rigidity
Talk 13: Monod Superrigidity
Talk 14: S-arithmetic groups
Moore Ergodicity
Fürstenberg Lemma
Chevalley's Theorem
Borel Density
Talk 7: Ergodic theory
Talk 8: Amenable groups
Talk 9: Algebraic groups & measures

Thanks for your attention!