Classifying space for proper actions for groups admitting a strict fundamental domain

Tomasz Prytuła

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joint work with Nansen Petrosyan

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Outline

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1 Classifying space for proper actions $\underline{E}G$

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2. Davis complex for a Coxeter group

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4. Applications

Classifying space for proper actions

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- $\underline{E}G$ always exists
- any two models for $\underline{E}G$ are G–homotopy equivalent

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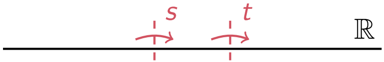
1. 

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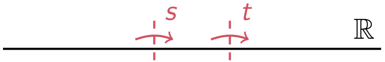

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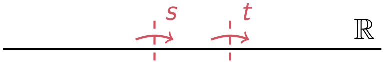
1. 

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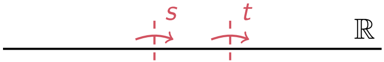
1. 

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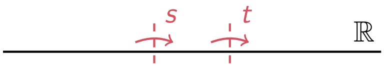
1. 

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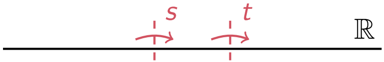

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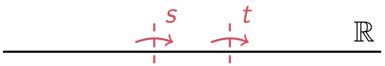
1. 

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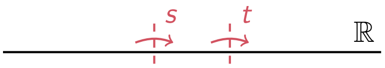
1. 

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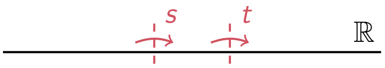

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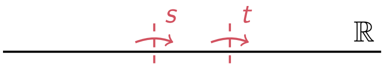
1. 

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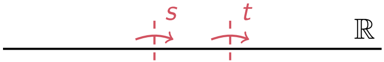
1. 

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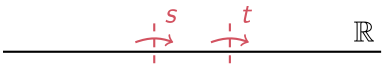
1. 

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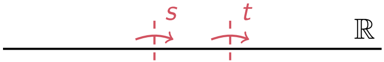
1. 

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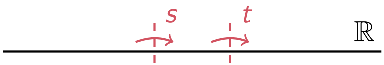
1. 

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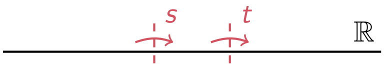
1. 

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Davis complex for a Coxeter group

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Right-Angled Coxeter groups

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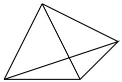

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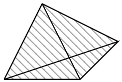

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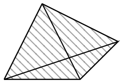

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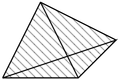

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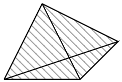

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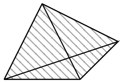

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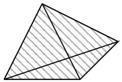

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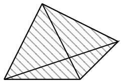

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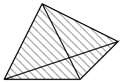

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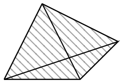

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Davis complex for a Coxeter group

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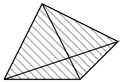

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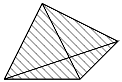

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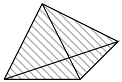

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$\underline{E}W = \Sigma_W = \Sigma$ - Davis complex

Davis complex for a Coxeter group

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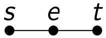
$CL' =$ 

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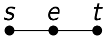
$CL' =$ 

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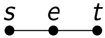
$CL' =$ 

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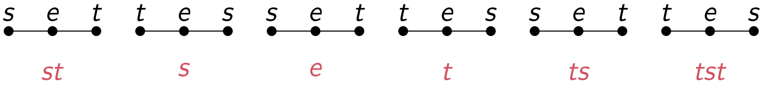

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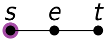
$CL' =$ 

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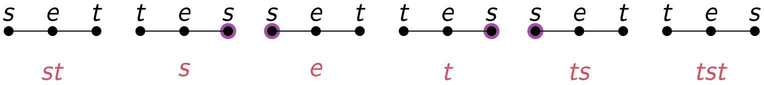

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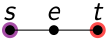
$CL' =$ 

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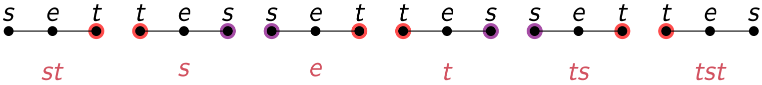

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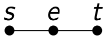
$CL' =$ 

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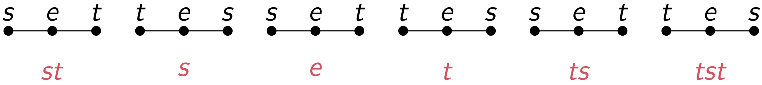

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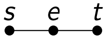
$CL' =$ 

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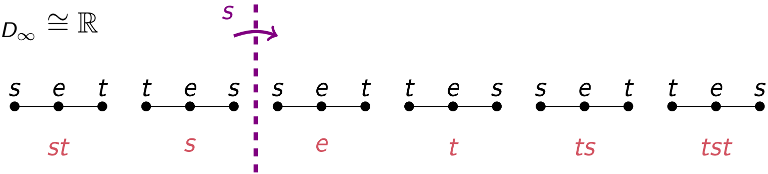

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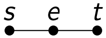
$CL' =$ 

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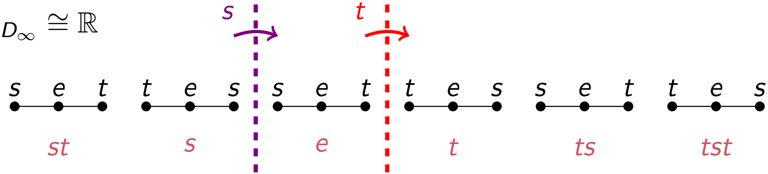

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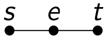
$CL' =$ 

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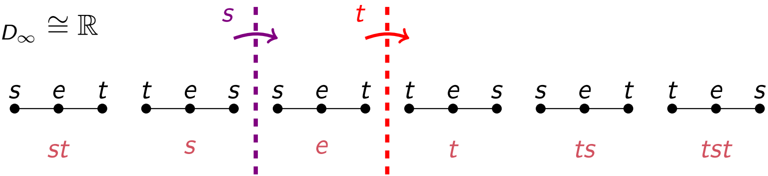

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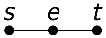
$CL' =$ 

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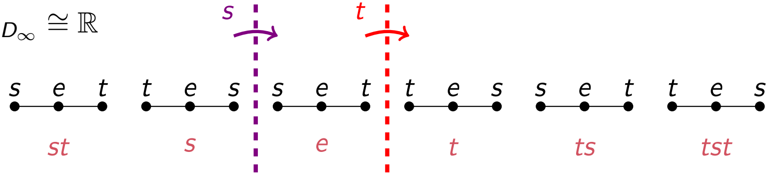

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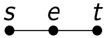
$CL' =$ 

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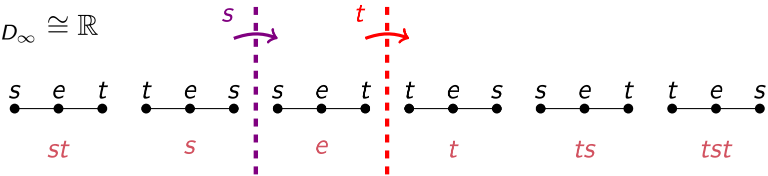

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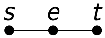
$CL' =$ 

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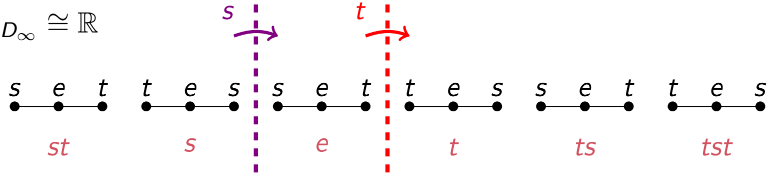

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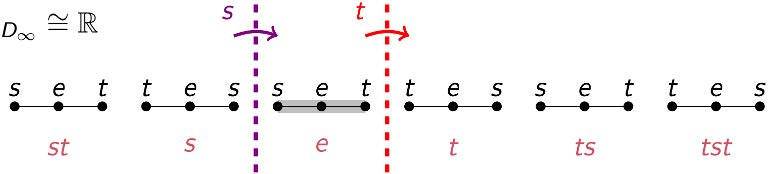

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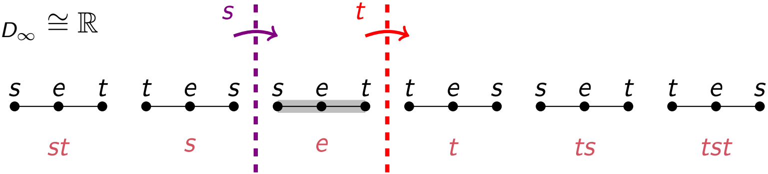

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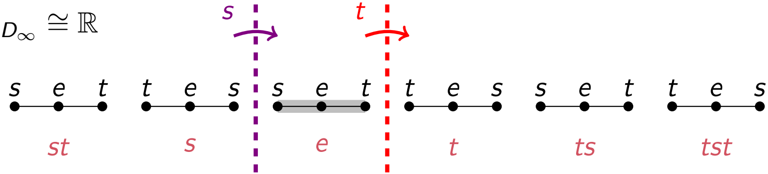

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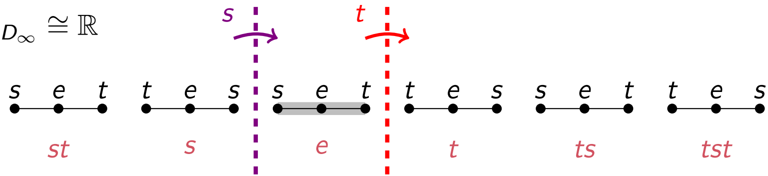

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Davis complex for a Coxeter group

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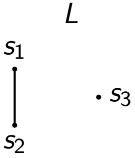

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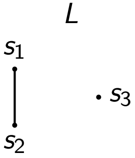

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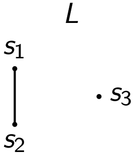

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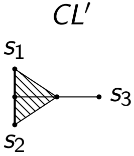

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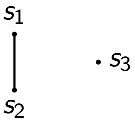

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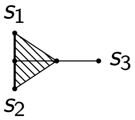

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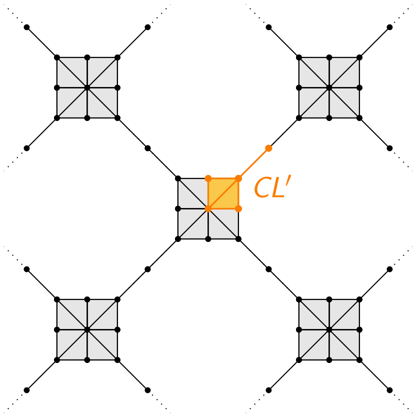

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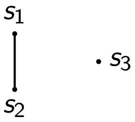

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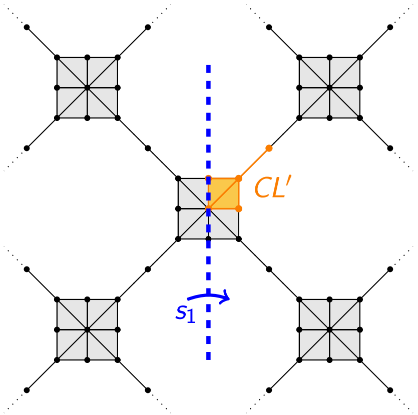

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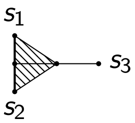

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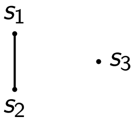

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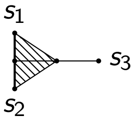

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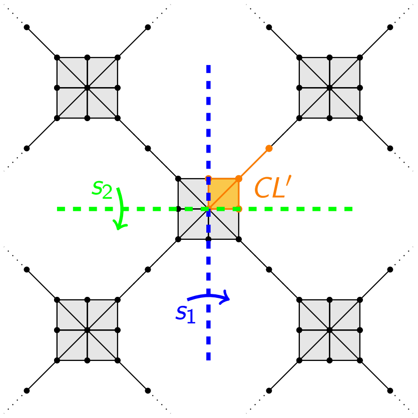

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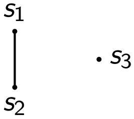

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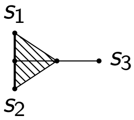

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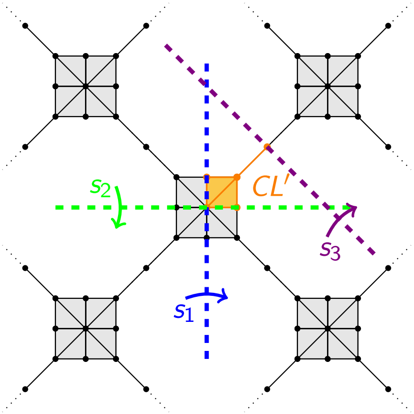

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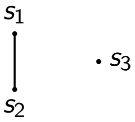

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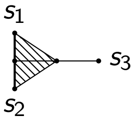

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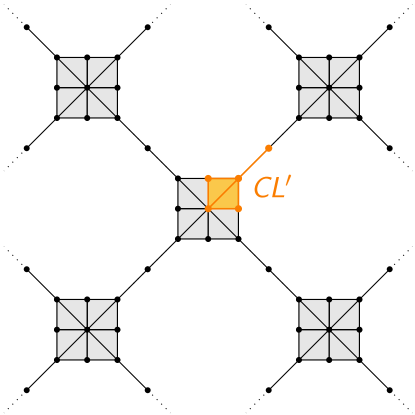

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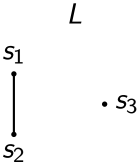

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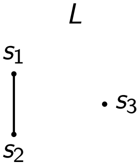

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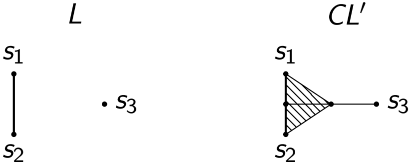

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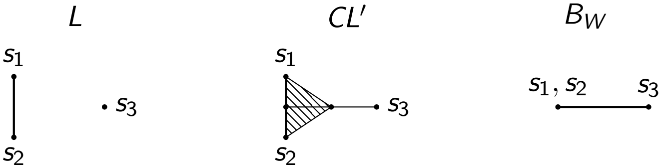

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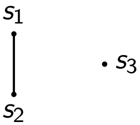

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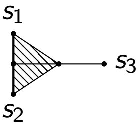

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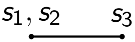

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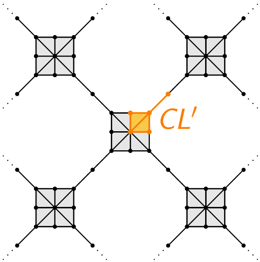

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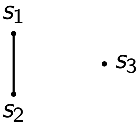

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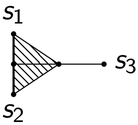

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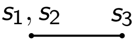

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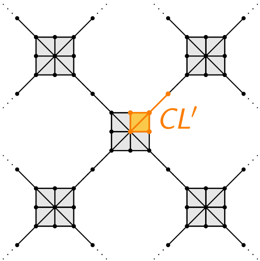

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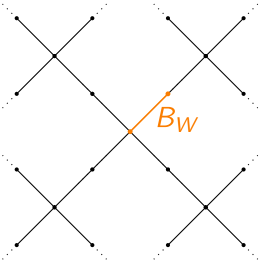

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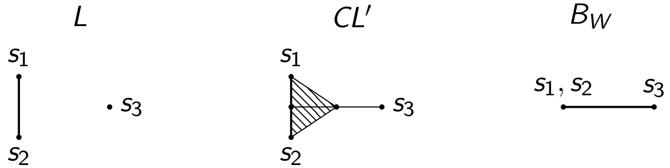

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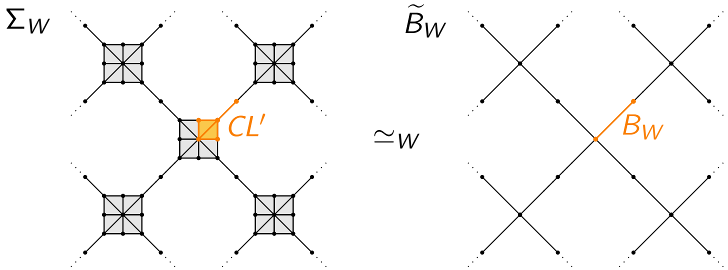

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