New construction of error-tolerant pooling designs

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Abstract We present a new class of error-tolerant pooling designs by constructing d^z-disjunct matrices associated with subspaces of a finite vector space.

Keywords Group testing, Nonadaptive algorithm, Pooling designs, d^z-disjunct matrix

1 Introduction

Combinatorial group testing has various practical applications [8], [9]. In the classical group testing model we have a set $[n] = \{1, \ldots, n\}$ of n items containing at most d defective items. The basic problem of group testing is to identify the set of all defective items with a small number of group tests. Each *group test*, also called a *pool*, is a subset of items. It is assumed that there is a testing mechanism that for each subset $A \subset [n]$ gives one of two possible outcomes : *negative* or *positive*. The outcome is positive if A contains at least one defective and is negative otherwise.

A group testing algorithm is called *nonadaptive* if all tests are specified without knowledge of the outcomes of other tests. Traditionally, a nonadaptive group testing algorithm is called a *pooling design*. Pooling designs have many applications in molecular biology, such as DNA screening, nonunique probe selection, gene detection, etc. (see [9], [10]).

A pooling design is associated with a $(0,1)-$ inclusion matrix $M = \{m_{ij}\}$, where the rows are indexed by tests $A_1, \ldots, A_t \subset [n]$, the columns are indexed by items $1, \ldots, n$, and $m_{ij} = 1$ if and only if $j \in A_i$. The major tool used for construction of pooling designs are $d-$*disjunct* matrices. Let M be a binary $t \times n$ matrix where the columns $C_1, \ldots, C_n$ are viewed as subsets of $[t] = \{1, \ldots, t\}$ represented by their characteristic vectors. Then M is called d–disjunct if no column is contained in the union of d others. The notion of $d-$disjunctness was introduced by Kautz and Singleton [14]. They proved that a d–disjunct matrix M can identify up to d defective items. $d-$disjunct matrices are also known as $d-$cover free families studied in extremal set theory [7].

The maximal d for which M is d–disjunct is called the degree of disjunctness and is denoted by d_{max}. Note that d–disjunctness of a pooling design is a sufficient, but not a necessary condition for identification of d defectives. However a $d-$disjunct pooling design has an advantage of a very simple decoding. Removing from the set of items all items in negative pools we get all defectives (see [9] for details).

A pooling design is called *error-tolerant* if it can detect/correct some errors in test outcomes. Biological experiments are known to be unreliable (see [9]), which, in fact, is a practical motivation for constructing efficient error-tolerant pooling designs.

For error correction in tests the notion of a *d^z-disjunct matrix* was introduced in [17]. A $d-$disjunct matrix is called *d^z-disjunct* if for any $d+1$ of its columns $C_{i_1},\dots,C_{i_{d+1}}$ we have $|C_{i_1}\setminus(C_{i_2}\cup\ldots\cup C_{i_{d+1}})|\geq z$. In fact, the d^1-disjunctness is simply the $d-$disjunctness. A d^z–disjunct matrix can detect $z-1$ errors and correct $\lfloor\frac{z-1}{2}\rfloor$ errors (see e.g. [10] or [9]). Constructions of d^z-disjunct matrices are given by many authors (see [2], [17], [18], [10]).

Most known constructions of d^z-disjunct matrices are matrices with a constant column weight. Let M be a binary $t\times n$ matrix with a constant column weight k and let s be the maximum size of intersection (number of common ones) between two different columns. Kautz and Singleton [14] observed that then M is d–disjunct with $d=\lfloor\frac{k-1}{s}\rfloor$. Moreover, for integers $0\leq s<k<t$ the maximum number $n(d,t,w)$ for which there exists such a disjunct matrix is upper bounded by

$$n(d,t,k)\leq\binom{t}{s+1}\Big/\binom{k}{s+1}. \tag{1.1}$$

Note that the columns of M considered as the family $\mathcal{F}$ of k–subsets of $[t]$ (called blocks) form an $(s+1,k,t)$–packing, that is each $(s+1)$–subset of $[t]$ is contained in at most one block of $\mathcal{F}$. Note also that equality in (1.1) is attained if and only if $\mathcal{F}$ is an $(s+1,k,t)$–Steiner system (each $(s+1)-$subset is contained in precisely one block).

Thus, packing designs can be used for construction of d–disjunct matrices. However, construction of good $(s+1,k,t)$–packings, in general, is known to be a difficult combinatorial problem. Several other constructions (see [9, Ch.3]) of disjunct matrices are also based on combinatorial structures or error correcting codes. We note that $(s+1,k,t)$–packings can also be described in terms of codes in the Johnson graph $J(n,k)$ (or Johnson scheme) with minimum distance $d_J=k-s$. It seems natural to try other distance regular graphs (see [4] for definitions), for construction of d–disjunct matrices, using the idea of packings.

In this paper we construct new error-tolerant pooling designs associated with finite vector spaces. In Section 2 we briefly review some known constructions of disjunct matrices based on partial orders and determine the degree of disjunctness for the construction proposed by Ngo and Du [18]. Our main results are stated and proved in Section 3. We present a construction of d^z–disjunct matrices based on packings in finite projective spaces. For certain parameters the construction gives better performance than previously known ones.

2 d^z–disjunct matrices from partial orders

Macula [16] proposed a simple direct construction of $d-$disjunct matrices. Given integers $1 \leq d < k < m$, let $M = (m_{ij})$ be an $\binom{m}{d} \times \binom{m}{k}$ matrix where the rows are indexed by elements of $\binom{[m]}{d}$, the columns are indexed by the elements of $\binom{[m]}{k}$, and $m_{ij} = 1$ if we have containment relation between the subsets corresponding to the ith row and the jth column, otherwise $m_{ij} = 0$. Note that each column has weight $\binom{k}{d}$ and each row has weight $\binom{m-d}{k-d}$. Macula showed that M is a d–disjunct matrix and $d_{max} = d$.

Similar constructions, using different posets, were given by several authors. Ngo and Du [18] extended Macula's construction to some geometric structures. In particular they considered the following construction of a $d-$disjunct matrix $M_q(m, d, k)$ associated with finite vector spaces. Let $GF(q)^m$ be the m–dimensional vector space over $GF(q)$. The set of all subspaces of $GF(q)^m$, called projective space, is denoted by $\mathcal{P}_q(m)$. Recall that $\mathcal{P}_q(m)$ ordered by containment is known as the poset of linear spaces (or linear lattice). Given an integer $0 \leq k \leq m$, the set of all k-dimensional subspaces (k–spaces for short) of $GF(q)^m$ is called a *Grassmannian* and denoted by $\mathcal{G}_q(m, k)$. Thus, we have $\bigcup_{0 \leq k \leq m} \mathcal{G}_q(m, k) = \mathcal{P}_q(m)$. A graph associated with $\mathcal{G}_q(m, k)$ is called the *Grassmann graph*, when two vertices (elements of $\mathcal{G}_q(m, k)$) V and U are adjacent iff $\dim(V \cap U) = k - 1$ (see [4] for more insight). It is known that the size of the Grassmannian $|\mathcal{G}_q(m, k)|$ is determined by the q-ary Gaussian coefficient $\begin{bmatrix} m \\ k \end{bmatrix}_q$; $k = 0, 1, \ldots, m$ $\left(\begin{bmatrix} m \\ 0 \end{bmatrix}_q \triangleq 1\right)$,

$$|\mathcal{G}_q(m, k)| = \begin{bmatrix} m \\ k \end{bmatrix}_q = \frac{(q^m - 1)(q^{m-1} - 1) \cdots (q^{m-k+1} - 1)}{(q^k - 1)(q^{k-1} - 1) \cdots (q - 1)}. \tag{2.1}$$

For integers $1 \leq r < k < m$, the $\begin{bmatrix} m \\ r \end{bmatrix}_q \times \begin{bmatrix} m \\ k \end{bmatrix}_q$ incidence matrix $M_q(m, r, k) = (m_{ij})$ is defined as follows. The rows and the columns are indexed by the elements of $\mathcal{G}_q(m, r)$ and $\mathcal{G}_q(m, k)$ (given in a fixed ordering), respectively, and $m_{ij} = 1$ if we have containment relation, otherwise $m_{ij} = 0$. Note that each column of $M_q(m, r, k)$ has weight $\begin{bmatrix} k \\ r \end{bmatrix}_q$ and each row has weight $\begin{bmatrix} m-r \\ k-r \end{bmatrix}_q$. Ngo and Du showed that $M_q(m, r, k)$ is an r–disjunct matrix. However D'yachkov et al. [10] observed that the degree of disjunctness of $M_q(m, r, k)$ can be much bigger than r. Moreover, the construction can in general tolerate many errors.

Theorem DHMVW [10]

For $k - r \geq 2$ and $d < \frac{q(q^{k-1}-1)}{q^{k-r}-1}$, the matrix $M_q(m, r, k)$ is d^z–disjunct with

$$z \geq \begin{bmatrix} k \\ r \end{bmatrix}_q - d \begin{bmatrix} k-1 \\ r \end{bmatrix}_q + (d-1) \begin{bmatrix} k-2 \\ r \end{bmatrix}_q. \tag{2.2}$$

The bound is tight for $d \leq q + 1$.

Note that the maximum number d in (2.2) for which $z > 0$ is $d = \frac{q(q^{k-1}-1)}{q^{k-r}-1}$. Thus, the theorem tells us that $d_{max} \geq \frac{q(q^{k-1}-1)}{q^{k-r}-1}$. In fact, we determine d_{max} for every $M_q(m, r, k)$.

Theorem 1 *For integers $1 \leq r < k < m$, the degree of disjunctness of $M_q(m, r, k)$ equals*

$$d_{max} = \frac{q(q^r - 1)}{q - 1}. \tag{2.3}$$

Proof. Let $V \in \mathcal{G}_q(m,k)$. We wish to determine the minimum size of a set of k–spaces which cover (contain) all r–spaces of V. Suppose $U_1,\dots,U_p \in \mathcal{G}_q(m,k)$ is a minimal covering of the r–spaces of V. Without loss of generality, we may assume that $\dim(U_i \cap V) = k-1$ for $i = 1,\dots,p$. Therefore, $W_1 = U_1 \cap V,\dots,W_p = U_p \cap V$ can be viewed as a set of hyperplanes of $\mathcal{P}_q(k)$ that cover all r–spaces of $\mathcal{P}_q(k)$. Let now $A_i \in \mathcal{P}_q(k)$ be the orthogonal space of W_i; $i = 1,\dots,p$. Thus, $\mathcal{A} = \{A_1,\dots,A_p\}$ is a set of one dimensional subspaces, that is points, in $\mathcal{P}_q(k)$. By the principle of duality, every $(k-r)$–space of $\mathcal{P}_q(k)$ contains an element of $\mathcal{A}$. To complete the proof we use the following result.

Theorem BB [3] *Let $\mathcal{A} \subset GF(q)^m \setminus \{0\}$ have a non-empty intersection with every $(k-r)$–space of $\mathcal{P}_q(k)$. Then $|\mathcal{A}| \geq (q^{r+1}-1)/(q-1)$, with equality if and only if $\mathcal{A}$ consists of $(q^{r+1}-1)/(q-1)$ points of an $(r+1)$–space of $\mathcal{P}_q(k)$.*

It is clear now that $d_{max} = (q^{r+1}-1)/(q-1)-1$. □

3 New construction

Our construction of a disjunct matrix M is based on packings in $\mathcal{P}_q(m)$. For integers $0 \leq s < k < m$, a subset $\mathcal{C} \subset \mathcal{G}(m,k)$ (with the elements called blocks) is called an $[s+1,k,m]_q$–packing if each $(s+1)$–space of $\mathcal{P}_q(m)$ is contained in at most one block of $\mathcal{C}$. This clearly means that $\dim(V \cap U) \leq s$ for every distinct pair $V, U \in \mathcal{C}$. $\mathcal{C}$ is called an $[s+1,k,m]_q$–Steiner structure if each $(s+1)$–space of $\mathcal{P}_q(m)$ is contained in precisely one block of $\mathcal{C}$. Let $N(m,k,s)$ denote the maximum size of an $[s+1,k,m]_q$–packing.
An equivalent definition of an $[s+1,k,m]_q$–packing can be given in terms of the subspace distance $d_S(V,U)$ defined (in general for any $V,U \in \mathcal{P}_q(m)$) by $d_S(V,U) = \dim V + \dim U - 2\dim(V \cap U)$ ([1], [15]). Then clearly $d_S(V,U) \geq 2(k-s)$ for every pair of elements $V,U \in \mathcal{C}$.
The following simple observation is an analogue of (1.1) for projective spaces. Let M be the incidence matrix of an $[s+1,k,m]_q$–packing $\mathcal{C}$ with $s \geq 1$, that is the $t \times n$ matrix where the rows (resp. columns) are indexed by the nonzero elements of $GF(q)^n$ (resp. by the blocks of $\mathcal{C}$) given in a fixed ordering.

Lemma 1 *(i) For $d \leq q^{k-s}$, the matrix M is d^z–disjunct with $z = q^k - 1 - d(q^s-1)$.*
(ii) The number of columns

$$n \leq N(m,k,s) \leq \begin{bmatrix} m \\ s+1 \end{bmatrix}_q \Big/ \begin{bmatrix} k \\ s+1 \end{bmatrix}_q \tag{3.1}$$

with both equalities if and only if $\mathcal{C}$ is an $[s+1,k,m]_q$–Steiner structure.

Proof. (i) By the definition of an $[s+1,k,m]_q$–packing, each $(s+1)$–space is contained in at most one k–space of $\mathcal{C}$. Therefore, any two columns in M have at most q^s-1 common ones. Hence, a column in M can be covered by at most $\lceil \frac{q^k-1}{q^s-1} \rceil > q^{k-s}$ other columns. Note that in case $s \mid k$, the space $GF(q)^k$ can be partitioned by s–spaces (see [5]) and $d_{max} = \frac{q^k-1}{q^s-1} - 1$.
(ii) Since the number of $(s+1)$–spaces contained in a k–space is $\begin{bmatrix} k \\ s+1 \end{bmatrix}_q$, we have the following

packing bound $N(m,k,s) \leq \left[{m \atop s+1}\right]_q / \left[{k \atop s+1}\right]_q$ (see [1], [20], [15]). The equality in (3.1) is attained iff we have a partition of all $(s+1)$–spaces by the blocks of $\mathcal{C}$. □

A challenging problem is to find Steiner structures in $\mathcal{P}_q(n)$. Note that no nontrivial Steiner structures, except for the case $s=0$ when we have a partition of $GF(q)^m$ by k–spaces, are known. Properties of Steiner structures in $\mathcal{P}_q(n)$, introduced in [1] are studied in [19].

Theorem WXS [20] (**KK** [15]) *Given integers $1<k<m$, there exists an explicit construction of an $[s+1,k,m]_q$–packing $\mathcal{C}$ with*

$$|\mathcal{C}| = \begin{cases} q^{(s+1)(m-k)} & \text{if } m \geq 2k,\ 0 \leq s < k \\ q^{k(s+1)} & \text{if } m < 2k,\ 0 \leq s < m-k. \end{cases} \tag{3.2}$$

The construction of such packings is based on Gabidulin codes [13] The explicit description (in terms of subspace codes) is given in [20] and in [15]. For completeness we describe this construction here (in terms of $[s+1,k,m]_q$–packings). Let $\mathbb{F}_q^{k\times r}$ denote the set of all $k\times t$ matrices over $GF(q)$. For $X,Y \in \mathbb{F}_q^{k\times r}$ the rank distance between X and Y is defined as $d_R(X,Y) = \text{rank}(X-Y)$. It is known that the rank-distance is a metric [13]. Codes in metric space $(\mathbb{F}_q^{k\times r}, d_R)$ are called rank-metric codes. It is known [13] that for a rank-metric code $\mathcal{C} \subseteq \mathbb{F}_q^{k\times r}$ with minimum distance $d_R(\mathcal{C})$ one has the Singleton bound $\log_q|\mathcal{C}| \leq \min\{k(r-d_R(\mathcal{C})+1), r(k-d_R(\mathcal{C})+1)\}$. Codes attaining this bound are called maximum-rank-distance codes (MRD). An important class of rank-metric codes are Gabidulin codes [13]. They are linear MRD codes, which exist for all parameters k,r and $d_R \leq \min\{k,r\}$. The construction of an $[s+1,k,m]_q$–packing from an MRD code is as follows. Consider the space $\mathbb{F}_q^{k\times(m-k)}$ $(m \geq k)$. Let first $m \geq 2k$. Then for any integer $0 \leq s \leq k$ there exists a Gabidulin code $\mathcal{C}_G \subset \mathbb{F}_q^{k\times(m-k)}$ of minimum distance $d_R = k-s$ and size $q^{(s+1)(m-k)}$. To each matrix $A \in \mathcal{C}_G$ we put into correspondence the matrix $[I_k|A] \in \mathbb{F}_q^{k\times m}$ (I_k is the $k\times k$ identity matrix). We define now the set of k–spaces $\mathcal{C}(m,k,s)_q = \{\text{rowspace}([I_k|A]) : A \in \mathcal{C}_G\}$. It can easily be observed now that $\dim(V\cap U) \leq s$ for all pairs $V,U \in \mathcal{C}(m,k,s)_q$. This means that $\mathcal{C}(m,k,s)_q$ is an $[s+1,k,m]_q$–packing with $|\mathcal{C}(m,k,s)_q| = |\mathcal{C}_G| = q^{(s+1)(m-k)}$. Similarly is described the $[s+1,k,m]_q$–packing $\mathcal{C}(m,k,s)_q$ for $m<2k$. Note that for our purposes the case $m \geq 2k$ is more important.

The following is a useful estimate for the Gaussian coefficients. A proof can be found in [6] (and in [15] for the case $q=2$).

Lemma 2 *For integers* $1 \leq k < m$ *we have*

$$q^{(m-k)k} < \begin{bmatrix} m \\ k \end{bmatrix}_q < \alpha(q)\cdot q^{(m-k)k}, \tag{3.3}$$

where $\alpha(2)=4$ *and* $\alpha(q)=\frac{q}{q-2}$ *for* $q \geq 3$.

Note that Lemma 2 in conjunction with Theorem WXS applied to our upper bound (3.1) shows that $\mathcal{C}(m,k,s)_q$ is nearly optimal:

$$|\mathcal{C}(m,k,s)_q| \leq |N(n,k,s)_q < \alpha(q)\cdot q^{(s+1)(m-k)} = \alpha(q)\cdot|\mathcal{C}(m,k,s)_q|.$$

Here actually $\lim \alpha(q) = 1$, as $q \to \infty$, yields asymptotic optimality.
Let $P(m,k,s)_q$ denote the incidence matrix of $\mathcal{C}(m,k,s)_q$. We summarize our findings in

Theorem 2 *Given integers $1 \le s < k \le \frac{1}{2}m$ and a prime power q, we have*
(i) $P(m,k,s)_q$ is a d–disjunct $t \times n$ matrix where $t = q^m - 1$, $n = q^{(s+1)(m-k)}$, $d = q^{k-s}$.
(ii) For any $d \le q^{k-s}$, the matrix $P(m,k,s)_q$ is d^z–disjunct with $z = q^k - 1 - d(q^s - 1)$.

Finally, we explain how good our construction is. Let $t(d,n)$ denote the minimum number of rows for a d–disjunct matrix with n columns. In the literature known are the bounds asymptotic in n

$$\Omega(1/d^2) \le \frac{\log n}{t(d,n)} \le O((\log d)/d^2) \tag{3.4}$$

(log is always of base 2). The lower bound is proved in [14], [11], [7] (see also [12], [9, ch.2]) using probabilistic methods. The upper bound is due to D'yachkov and Rykov [11].

Next we compare our construction with the construction in Ngo and Du [18], described in Section 2 (both constructions we take over $GF(q)$). In their construction we have $n \le \alpha(q)q^{(m-k)k}$, $t \ge q^{(m-r)r}$ (Lemma 2), $d = \frac{q(q^r-1)}{q-1}$ (Theorem 2), and rate $(\log n)/t$.

For the parameters in our construction we use the notation n_0, k_0, t_0, d_0. Thus, $n_0 = q^{(s+1)(m_0-k_0)}$, $t_0 = q^{m_0} - 1$, $d_0 \ge q^{k_0-s}$. We put $m_0 = m$, $k_0 = k$, $s = k - r - 1$. Then we have $n_0 = q^{(k-r)(m-k)}$, $t_0 = q^m - 1$, $d_0 \ge q^{r+1} > d$, and rate $(\log n_0)/t_0$. A simple calculation shows that $(\log n_0)/t_0$ exceeds $(\log n)/t$ by a factor $q^{m(r-1)-r^2} \cdot \frac{k-r}{k+1}$.

Let us take now in our construction $q = 2$, $m = 2k$. Then we have $d = 2^{k-s}$, $t = 2^{2k} - 1$, $n = 2^{(s+1)k}$ and hence

$$\frac{\log n}{t} > \frac{(s+1)k}{2^{2k}} > \frac{s+1}{2^{2s}} \cdot \frac{\log d}{d^2}.$$

Corollary 1 *Given integer $s \ge 1$, our construction gives a class of d–disjunct $t \times n$ matrices with parameters $d = 2^{k-s}$, $t = 2^{2k}$, $n = 2^{(s+1)k}$ attaining the upper bound in (3.4), that is rate $(\log n)/t = \Omega((\log d)/d^2)$.*

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