ISSN 0032-9460, Problems of Information Transmission, 2012, Vol. 48, No. 2, pp. 173–181. © Pleiades Publishing, Inc., 2012.
Original Russian Text © R. Ahlswede, C. Deppe, V.S. Lebedev, 2012, published in Problemy Peredachi Informatsii, 2012, Vol. 48, No. 2, pp. 100–109.

Finding One of D Defective Elements in Some Group Testing Models

R. Ahlswede†, C. Deppe[a1], and V. S. Lebedev[b2]

[a] *Department of Mathematics, University of Bielefeld, Germany*
`cdeppe@mathematik.uni-bielefeld.de`
[b] *Kharkevich Institute for Information Transmission Problems,*
Russian Academy of Sciences, Moscow
`lebed37@iitp.ru`

Received May 10, 2011

Abstract—In contrast to the classical goal of group testing, we consider the problem of finding m defective elements out of D ($m \leq D$). We analyze two different test functions. We give adaptive strategies and present lower bounds for the number of tests and show that our strategy is optimal for $m = 1$.

DOI: 10.1134/S0032946012020068

1. INTRODUCTION

Group testing is of interest in many applications, for instance, in molecular biology. For an overview of results and applications, we refer the reader to [1, 2].

In [3] the authors considered the problem of finding one defective element in a finite set where an element i is defective with probability p_i. The case where all probabilities are identical appears already in [4].

In [5] the authors considered the problem of finding at least k nondefective elements. Their study was motivated by a practical problem of an electronic company. The production department of the company requires 10^6 nondefective electronic chips for their production process. There is a method for testing a pool of chips. They buy chips of 99% quality (a chip is defective with probability 0.01) and want to find many nondefective elements in a small number of group tests.

We will consider a combinatorial version of this problem. Thus, it is required to find m out of D defective elements. This study was motivated by [6, 7]. We denote by $[N] := \{1, 2, \ldots, N\}$ the set of elements, by $\mathcal{D} \subset [N]$ the set of defective elements, by $D = |\mathcal{D}|$ its cardinality, and by $[i, j]$ the set of integers $\{x \in \mathbb{N} : \ i \leq x \leq j\}$. Throughout the paper we consider the worst case analysis.

The classical group testing problem consists in finding an unknown subset $\mathcal{D}$ of all defective elements in $[N]$.

For a subset $\mathcal{S} \subset [N]$, a test $t_{\mathcal{S}}$ is a function $t_{\mathcal{S}} \colon 2^{[N]} \to \{0, 1\}$ defined by

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| = 0, \\ 1 & \text{otherwise.} \end{cases} \tag{1}$$

---

[1] Supported in part by the German Research Council (DFG), project no. AH46/6-1 “Advances in Search and Sorting.”

[2] Supported in part by the Russian Foundation for Basic Research, project no. 12-01-00905-a, and German Research Council (DFG), project no. AH46/6-1 “Advances in Search and Sorting.”

We define search strategies as in [8]. In classical group testing, a strategy is called successful if we can *uniquely determine* $\mathcal{D}$. Here we call a strategy successful if we can find m elements of $\mathcal{D}$. Recall the concepts of adaptive and nonadaptive strategies.

Strategies are called adaptive if the kth test is determined by results of the first $k-1$ tests. Strategies where all tests are chosen independently are said to be nonadaptive.

Let f be a function $f\colon [0,N] \to \mathbb{R}^+$. We define *general group tests with density* as functions $t_{\mathcal{S}}\colon 2^{[N]} \to \{0,1\}$ of the form

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| < f(|\mathcal{S}|), \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq f(|\mathcal{S}|). \end{cases} \tag{2}$$

In [7] the case $f(|\mathcal{S}|) = \alpha|\mathcal{S}|$ is considered. The authors assume that a lower bound on the cardinality of $\mathcal{D}$ is known. *The goal is finding $m \leq D$ defective elements.*

In *majority group testing* (defined in [9] and, more generally, in [10]), we have two functions

$$f_1, f_2\colon \{0,1,\ldots,N\} \to \mathbb{R}^+$$

which define weights on the numbers D of defective elements such that

$$f_1(D) \leq f_2(D) \quad \text{for all} \quad D \in [0,1,\ldots,N].$$

We describe the structure of the tests $t_{\mathcal{S}}\colon 2^{[N]} \to \{0,1,\{0,1\}\}$ as follows:

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| < f_1(D), \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq f_2(D), \\ \{0,1\} & \text{otherwise} \\ & \text{(meaning that the result can be 0 or 1 arbitrarily).} \end{cases} \tag{3}$$

In [10] it is assumed that the searcher does not know the cardinality of $\mathcal{D}$ but knows some upper bound. In majority group testing *it is not always possible to find the set $\mathcal{D}$ of all defective elements* (see [10, 11]). In general, one can *find a family $\mathbb{F}$ of sets which contains $\mathcal{D}$*. This family depends on f_1, f_2, $\mathcal{D}$, and the strategy used. In this case we say that a strategy is successful if we can find a family $\mathbb{F}$ with the smallest possible size.

Now we put ideas of these two models together so that there are two functions

$$f_1, f_2\colon [0,N] \times [0,N] \to \mathbb{R}^+$$

with $f_1(D,S) \leq f_2(D,S)$ for all values of D and S.

We define a test $t_{\mathcal{S}}\colon 2^{[N]} \to \{0,1,\{0,1\}\}$ as follows:

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| < f_1(D,|\mathcal{S}|), \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq f_2(D,|\mathcal{S}|), \\ \{0,1\} & \text{otherwise} \\ & \text{(meaning that the result can be 0 or 1 arbitrarily).} \end{cases} \tag{4}$$

For this test function, denote by $n(N,D,m)$ the minimal number of tests for finding m defective elements.

The following lower bound for the minimal number of test is a generalization of a theorem in [7], where this lower bound is given for the case of

$$f_1(D,|\mathcal{S}|) = f_2(D,|\mathcal{S}|) = \alpha|\mathcal{S}|.$$

This bound is valid for any binary test function (i.e., a test function taking values 0 or 1).

Theorem 1. *We have*

$$n(N, D, 1) \geq \lceil \log(N - D + 1) \rceil.$$

Proof. Let us assume that we have a successful strategy s which finds a defective element with $n = n(N, D, 1)$ tests and $n < \lceil \log(N - D + 1) \rceil$.

Depending on results of n tests, we have at most 2^n different possible results for a defective element; we denote them by $\mathcal{E}$. We have by assumption that

$$|\mathcal{E}| \leq 2^n < N - D + 1.$$

Therefore, $|[N] \setminus \mathcal{E}| > D - 1$, and there exists a set $\mathcal{F} \subset [N] \setminus \mathcal{E}$ with $|\mathcal{F}| = D$. Now we consider the case of $\mathcal{D} = \mathcal{F}$. Then it is obvious that using strategy s we cannot find any defective element in n tests. △

We consider the following special cases of this test model: $f = f_1 = f_2$, and D is known.

Threshold group testing without gap: $f(D, |\mathcal{S}|) = u$; thus,

$$t_S(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| < u, \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq u. \end{cases} \tag{5}$$

Group testing with density tests: $f(D, |\mathcal{S}|) = \alpha |\mathcal{S}|$ for all values. Thus,

$$t_S(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| < \alpha |\mathcal{S}|, \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq \alpha |\mathcal{S}|. \end{cases} \tag{6}$$

For all these test functions we consider the adaptive model with the goal of finding one defective element.

In Section 1 we consider test function (1) (classical case) and give an optimal strategy for finding one out of D defectives with $\lceil \log(N - D + 1) \rceil$ tests. In Section 2 we give strategies for test function (5) (threshold case) and show that the strategy is optimal for $m = 1$. Furthermore, we combine the strategy with that given in [12] for finding m elements. In Section 3 we give a strategy for test function (6) and give some remarks on nonadaptive group testing.

2. CLASSICAL TEST FUNCTION

In this section we use test function (1). We assume that D $(0 < D < N)$ is known. Our goal is finding m defective elements.

We denote by $n_{(\mathrm{Cla})}(N, D, m)$ the minimal number of tests (1) required for finding m defective elements.

Proposition 1. *We have*

$$n_{(\mathrm{Cla})}(N, D, 1) \leq \lceil \log(N - D + 1) \rceil.$$

Proof. We give a strategy which needs $\lceil \log(N - D + 1) \rceil$ tests. We know that the set $\mathcal{S}_0 = \{D, D+1, \ldots, N\}$ contains at least one defective element. Thus, we start with the test set $\mathcal{S}_1 \subset \mathcal{S}_0$ of size $\left\lfloor \dfrac{N - D + 1}{2} \right\rfloor$. If the test is positive, then at least one defective element is in $\mathcal{S}_1$; otherwise, at least one defective element is in $\mathcal{S}_0 \setminus \mathcal{S}_1$. Therefore, depending on the test result, we replace $\mathcal{S}_0$ by either $\mathcal{S}_1$ or $\mathcal{S}_0 \setminus \mathcal{S}_1$ and iterate the procedure. With this method we find one defective element in $\lceil \log(N - D + 1) \rceil$ tests. △

Proposition 1 and Theorem 1 imply the following result.

Corollary 1. *We have*

1. $n_{(\mathrm{Cla})}(N, D, 1) = \lceil \log(N - D + 1) \rceil$,
2. $n_{(\mathrm{Cla})}(N, D, m) \leq m \lceil \log(N - D + 1) \rceil$.

Remark 1. If we do not know D but know that $1 \leq D' < D'' < N$ with $D' \leq D \leq D''$, then we need $\lceil \log(N - D' + 1) \rceil$ tests for finding one defective element.

3. THRESHOLD TEST FUNCTION WITHOUT GAP

The threshold testing

$$t_S(\mathcal{D}) = \begin{cases} 0 & \text{if } |\mathcal{S} \cap \mathcal{D}| \leq l, \\ 1 & \text{if } |\mathcal{S} \cap \mathcal{D}| \geq u, \end{cases} \tag{7}$$

was introduced in [11]. The gap between the upper and lower thresholds is defined to be $g = u - l - 1$. Now we will consider test function (5), which corresponds to the case of no gap ($g = 0$). One easily sees that $u = l - 1$ in this case. First we assume that we know D.

We denote by $n_{(\mathrm{Thr})}(N, D, u, m)$ the minimal number of tests (5) for finding m defective elements if we have N elements with D defectives and $f(D, |\mathcal{S}|) = u$.

Our first goal is finding one defective element.

Proposition 2. *If* $D \geq u$, *then*

$$n_{(\mathrm{Thr})}(N, D, u, 1) \leq \lceil \log(N - D + 1) \rceil;$$

otherwise, finding any defective element is impossible.

Proof. We give a strategy which needs $\lceil \log(N - D + 1) \rceil$ tests. The idea of the proof is to partition the set of N elements into subsets

$$\mathcal{I}_1 = [1, u - 1], \qquad \mathcal{I}_2 = [u, N - D + u], \qquad \mathcal{I}_3 = [N - D + u + 1, N].$$

In $\mathcal{I}_2$ there is of course at least one defective element, because the union of the two other subsets has cardinality $D - 1$. We can find a defective element in $\mathcal{I}_2$ using the following strategy with $\lceil \log(N - D + 1) \rceil$ tests.

We start with the test set

$$\mathcal{S}_1 = \left\{ 1, \ldots, u - 1, u, \ldots, (u - 1) + \left\lceil \frac{m(1)}{2}(N - D + 1) \right\rceil \right\},$$

where $m(1) = 1$.

Inductively, we set

$$m(j) = \begin{cases} 2m(j-1) - 1 & \text{if } t_{S_{j-1}}(\mathcal{D}) = 1, \\ 2m(j-1) + 1 & \text{if } t_{S_{j-1}}(\mathcal{D}) = 0, \end{cases}$$

and

$$\mathcal{S}_j = \left\{ 1, \ldots, u - 1, u, u + 1, \ldots, (u - 1) + \left\lceil \frac{m(j)}{2^j}(N - D + 1) \right\rceil \right\}.$$

After $\lceil \log(N - D + 1) \rceil$ tests we can find an i such that $t_{[1,i]} = 1$ and $t_{[1,i-1]} = 0$, because it is clear that $t_{[1,u-1]} = 0$ and $t_{[1,N-D+u]} = 1$. Thus, using this strategy, we find a defective element at the position i. $\triangle$

From Theorem 1 and Proposition 2 we get the following result.

Theorem 2. *If* $D \geq u$, *then*

$$n_{(\mathrm{Thr})}(N, D, u, 1) = \lceil \log(N - D + 1) \rceil.$$

The strategy can be generalized to the case of finding m defective elements.

Proposition 3. *Let* $D \geq m$. *Then*

$$n_{(\mathrm{Thr})}(N, D, u, m) \leq m \lceil \log(N - D + 1) \rceil.$$

Proof. We apply the strategy used in Proposition 2 for finding one defective element. We use the ordered set $[N]$ and denote by $\pi_j(i)$ the jth position before the ith test. We set $\pi_j(1) = j$. In the first round we apply the strategy of Proposition 2 and find a defective element d_1. Then we define

$$\pi_j(2) = \begin{cases} d_1 & \text{if } j = 1, \\ 1 & \text{if } j = d_1, \\ j & \text{if } j \notin \{1, d_1\} \end{cases}$$

(i.e., we exchange the elements at the positions d_1 and 1) and apply the same strategy with $\lceil \log(N - D + 1) \rceil$ tests to find a defective element d_2 for the new set $\{\pi_1(2), \pi_2(2), \ldots, \pi_N(2)\}$. Now we exchange the elements at the positions d_2 and 2 and iterate this procedure, exchanging after every round the elements at the positions d_j and j, until we find a defective element d_u. From now on we exchange the defective element at the position d_j with the element at the position $N - D + 1 + j$. In total, after m iterations, we find m defectives. △

Remark 2. If we have already found $u - 1$ defective elements, we can use any classical group testing strategy to find the remaining $D - u + 1$ defectives in the set of $N - u + 1$ unknown elements by adding the $u - 1$ defective elements to each test.

We apply this improvement if we want to find all defective elements, using the following result of [12]:

$$n_{(\mathrm{Cla})}(N, D, D) \leq \left\lceil \log \binom{N}{D} \right\rceil + D - 1.$$

We proceed as follows. After $u - 1$ rounds in the proof of Proposition 3, we use the strategy of [12] for the remaining $N - u + 1$ elements with $D - u + 1$ defectives, and then we get a total of

$$T(u) = (u - 1) \lceil \log(N - D + 1) \rceil + \left\lceil \log \binom{N - u + 1}{D - u + 1} \right\rceil + D - u + 1$$

tests. This gives the following upper bound.

Theorem 3. *We have*

$$n_{(\mathrm{Thr})}(N, D, u) \leq T(u).$$

If D is unknown, we can take one test with all elements. Then, if the answer is negative, we cannot find any defective element. If the answer is positive, we know that $D \geq u$.

So we are interesting in the case where we do not know D, but we have $u \leq D \leq N$.

If D is unknown, we denote by $n_{(\mathrm{Thr})}(N, u, m)$ the minimal number of tests (5) required for finding m defective objects in the worst case if we have N elements and $f(|\mathcal{D}|, |\mathcal{S}|) = u$ for all values. In this case there is the following estimate.

Lemma 1. *We have*

$$n_{(\mathrm{Thr})}(N, u, m) \leq m \lceil \log(N - u + 1) \rceil.$$

Proof. If D is unknown, a similar idea works as in the proof of Proposition 3. We give a strategy which needs $m\lceil\log(N-u+1)\rceil$ tests. We use m adaptive rounds and start with a test set

$$\mathcal{S}_1 = \left\{1, \ldots, u-1, u, \ldots, (u-1) + \left\lceil \frac{m(1)}{2}(N-u+1) \right\rceil \right\},$$

where $m(1) = 1$.

For $j \le \lceil\log(N-u+1)\rceil$ we set

$$m(j) = \begin{cases} 2m(j-1) - 1 & \text{if } t_{S_{j-1}}(\mathcal{D}) = 1, \\ 2m(j-1) + 1 & \text{if } t_{S_{j-1}}(\mathcal{D}) = 0, \end{cases}$$

and

$$\mathcal{S}_j = \left\{1, \ldots, u-1, u, \ldots, (u-1) + \left\lceil \frac{m(j)}{2^j}(N-u+1) \right\rceil \right\}.$$

First we find one defective element d_1 using $\lceil\log(N-u+1)\rceil$ tests. Now instead of the set $\{1, 2, \ldots, N\}$ we use the set $\{\pi_1, \pi_2, \ldots, \pi_N\}$, where

$$\pi_j = \begin{cases} d_1 & \text{if } j = 1, \\ 1 & \text{if } j = d_1, \\ j & \text{if } j \notin \{1, d_1\}, \end{cases}$$

and then continue as before with $\lceil\log(N-u+1)\rceil$ tests and find the defective element d_2 for the new set $\{\pi_1, \pi_2, \ldots, \pi_N\}$. Then we iterate this procedure until we find $u-1$ defectives. Then we know that the remaining $D-u+1$ defectives objects are in the set $[u, N]$. These defectives can be found in $(m-u+1)$ rounds with $\lceil\log(N-u+1)\rceil$ tests. △

4. DENSITY TESTS

Test model (6) was considered in [7].

Let $n_{(\mathrm{Den})}(N, D, m, \alpha)$ be the minimal number of tests (6) required for finding m defective elements if we have N elements with D defectives. In [7] the authors obtain the following bounds for $n_{(\mathrm{Den})}(N, D, m, \alpha)$ assuming that $D \ge \alpha N$:

$$\lceil\log N\rceil + \max_{N' \le 2m/\alpha} n_{(\mathrm{Den})}(N', m, m, \alpha) \ge n_{(\mathrm{Den})}(N, D, m, \alpha), \tag{8}$$

$$\lceil\log N\rceil \ge n_{(\mathrm{Den})}(N, D, 1, \alpha). \tag{9}$$

In general they show that

$$\log(N-D+1) \le n_{(\mathrm{Den})}(N, D, 1, \alpha). \tag{10}$$

Test model (6) gives the same result as test model (1) if the size of the test set is smaller than $1/\alpha$. In the strategy given in the proof of Proposition 1, the biggest test set $\mathcal{S}_0$ has cardinality $\left\lfloor \frac{N-D+1}{2} \right\rfloor$. If $|S_0|$ in test model (6) is smaller than $1/\alpha$, we can apply the strategy and get

$$n_{(\mathrm{Den})}(N, D, 1, \alpha) \le \lfloor\log(N-D+1)\rfloor.$$

This is the case if $D \ge N + 1 - \dfrac{2}{\alpha}$. Therefore, we obtain the following result.

Proposition 4. *Let* $D \geq N + 1 - \dfrac{2}{\alpha}$. *Then*

$$n_{(\mathrm{Den})}(N, D, 1, \alpha) = \lceil \log(N - D + 1) \rceil.$$

Now we will improve the result. We will give a strategy which is optimal for $D \geq \alpha N$ (it needs $\lceil \log(N - D + 1) \rceil$ questions).

We define

$$s_i = \left\lceil \frac{2^{n-i} - 1}{1 - \alpha} \right\rceil,$$

where $i = 1, 2, \ldots, n - 1$, and $s_n = 1$. For a given D we choose the largest n such that

$$D > \sum_{i=1}^{n} s_i - 2^n + 1. \tag{11}$$

We consider test sets

$$\mathcal{S}_i = \{a_i + 1, a_i + 2, \ldots, a_i + s_i\}$$

for $i = 1, \ldots, n$, where $a_1 = 0$ and

$$a_i = \begin{cases} a_{i-1} + s_{i-1} & \text{if } t_{S_{i-1}}(\mathcal{D}) = 0, \\ a_{i-1} & \text{if } t_{S_{i-1}}(\mathcal{D}) = 1. \end{cases} \tag{12}$$

Note that $|S_i| = s_i$.

Lemma 2. *If* $t_{S_{n-j}}(\mathcal{D}) = 1$, *then we can find one defective element after* n *tests.*

Proof. We proceed by induction on j. The case of $j = 0$ is obvious. Let us also consider the case of $j = 1$ (to show the idea of the strategy). We have $s_{n-1} = \left\lceil \dfrac{1}{1 - \alpha} \right\rceil$ and $t_{S_{n-1}}(\mathcal{D}) = 1$. Then

$$s_{n-1} - 2 < \alpha s_{n-1} \leq s_{n-1} - 1.$$

Thus, in the set S_{n-1} we have no more than one nondefective element. If $t_{S_n}(\mathcal{D}) = 1$, this gives us a defective element; otherwise ($t_{S_n}(\mathcal{D}) = 0$), we can take any element from $S_n \setminus S_{n-1}$.

We assume that the statement is proved for $j - 1$. Let $t_{S_{n-j}}(\mathcal{D}) = 1$; then by the induction hypothesis we may assume that $t_{S_{n-i}}(\mathcal{D}) = 0$ for all $0 \leq i < j$.

Thus, the number of nondefective elements in S_{n-j} is not greater than $2^j - 1$, since $t_{S_{n-i}}(\mathcal{D}) = 1$ and

$$s_{n-j} - 2^j < \alpha s_{n-j} \leq s_{n-j} - 2^j + 1.$$

On the other hand, the number of nondefective elements in S_{n-i} for all $0 \leq i < j$ is greater than or equal to 2^i, since $t_{S_{n-i}}(\mathcal{D}) = 0$. Thus, all elements in $S_{n-j} \setminus \bigcup\limits_{i<j} S_{n-i}$ are defective.

The set $S_{n-j} \setminus \bigcup\limits_{i<j} S_{n-i}$ is nonempty, because for any k and α, $0 < \alpha < 1$, we have

$$1 + \sum_{i=1}^{k} \left\lceil \frac{2^i - 1}{1 - \alpha} \right\rceil < 1 + k + \sum_{i=1}^{k} \frac{2^i - 1}{1 - \alpha} = 1 + k + \frac{2^{k+1} - k - 2}{1 - \alpha} < \frac{2^{k+1} - 1}{1 - \alpha}. \quad \triangle$$

Theorem 4. *Let* (11) *be fulfilled, and let* $N \leq 2^n + D - 1$. *Then after* n *tests of the strategy given above we find one defective element.*

Proof. Consider the tests sets defined in (12). If for some i we have $t_{S_i}(\mathcal{D}) = 1$, then the theorem follows by Lemma 2. If $t_{S_i}(\mathcal{D}) = 0$ for all $i = 1, 2, \ldots, n$, then we denote by c_i the number of nondefective elements in S_i. The number of defectives in S_i is $s_i - c_i$. Thus, we have $s_i - c_i < \alpha s_i$, and hence $c_i \geq 2^i$.

In total, the number of nondefective elements is not less than $2^n - 1$ and, since

$$N - D = 2^n - 1,$$

we can take any element of $[N] \setminus \bigcup\limits_{t=1}^{n} S_t$. Note that if

$$N < 2^n + D - 1,$$

then there is an i with $t_{S_i}(\mathcal{D}) = 1$. △

Corollary 2. *If* $D \geq \alpha N$, *then*

$$n_{(\mathrm{Den})}(N, D, 1) = \lceil \log(N - D + 1) \rceil.$$

Proof. By (10) we have

$$D > \sum_{k=0}^{n-1} \left(\left\lceil \frac{2^k - 1}{1 - \alpha} \right\rceil - 2^k \right).$$

Note that

$$n - 1 + \sum_{k=1}^{n-1} \left(\frac{2^k - 1}{1 - \alpha} - 2^k \right) = \frac{\alpha}{1 - \alpha}(2^n - n - 1).$$

If we take

$$D > \frac{\alpha}{1 - \alpha}(2^n - n - 1)$$

and

$$N < 2^n + \frac{\alpha}{1 - \alpha}(2^n - n - 1) - 1,$$

then

$$\frac{N}{D} < \frac{1 - \alpha}{\alpha} + 1 + \frac{(1 - \alpha)n}{\alpha(2^n - n - 1)}.$$

Thus, if $D \geq \alpha N$, we can apply Theorem 4. △

Remarks (nonadaptive case).

In [6] it is shown that for test (1), if D is unknown, one needs N tests for finding one defective element or claiming that there are no defective elements. If D is known, we can test $N - D$ elements to find one defective element or we can use a $(D, 1)$ cover-free code for finding all elements and thereby one element as well.

A nonadaptive model for majority group testing was considered in [9, 10]. The goal in these papers was finding all defective elements.

Results of [13] for row-weighted cover-free codes can be used to get strategies for test (6) if the number of defectives is known.

REFERENCES

1. Du, D.-Z. and Hwang, F.K., *Combinatorial Group Testing and Its Applications*, Singapore: World Sci., 2000, 2nd ed.
2. Du, D.-Z. and Hwang, F.K., *Pooling Designs and Nonadaptive Group Testing. Important Tools for DNA Sequencing*, Hackensack, NJ: World Sci., 2006.

3. Garey, M.R. and Hwang, F.K., Isolating a Single Defective Using Group Testing, *J. Amer. Statist. Assoc.*, 1974, vol. 69, no. 345, pp. 151–153.
4. Kumar, S. and Sobel, M., Finding a Single Defective in Binomial Group-Testing, *J. Amer. Statist. Assoc.*, 1971, vol. 66, no. 336, pp. 824–828.
5. Bar-Lev, S.K., Boneh, A., and Perry, D., Incomplete Identification Models for Group-Testable Items, *Naval Res. Logistics*, 1990, vol. 37, no. 5, pp. 647–659.
6. Katona, G.O.H., Finding at Least One Defective Element in Two Rounds, in *Search Methodologies (Dagstuhl Seminar 09281, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Germany)*, 2009.
7. Gerbner, D., Keszegh, B., Pálvölgyi, D., and Wiener, G., Search with Density Tests, in *Search Methodologies II (ZiF Workshop, Bielefeld, Germany, 2010)*, pp. 33.
8. Ahlswede, R. and Wegener, I., *Suchprobleme*, Stuttgart: Teubner, 1979.
9. Lebedev, V.S., Separating Codes and a New Combinatorial Search Model, *Probl. Peredachi Inf.*, 2010, vol. 46, no. 1, pp. 3–8 [*Probl. Inf. Trans.* (Engl. Transl.), 2010, vol. 46, no. 1, pp. 1–6].
10. Ahlswede, R., Deppe, C., and Lebedev, V.S., Threshold and Majority Group Testing, to appear in *Lect. Notes Comp. Sci.*
11. Damaschke, P., Threshold Group Testing, *General Theory of Information Transfer and Combinatorics*, Ahlswede, R., Bäumer, L., Cai, N., Aydinian H.K., Blinovsky, V., Deppe, C., and Mashurian, H., Eds., Lect. Notes Comp. Sci., vol. 4123, Berlin: Springer, 2006, p. 707–718.
12. Hwang, F.K., A Method for Detecting All Defective Members in a Population by Group Testing, *J. Amer. Statist. Assoc.*, 1972, vol. 67, no. 339, pp. 605–608.
13. D'yachkov, A.G. and Rykov, V.V., Optimal Superimposed Codes and Designs for Renyi's Search Model, *J. Statist. Plann. Inference*, 2002, vol. 100, no. 2, pp. 281–302.