

# SEARCH PROBLEMS

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## Preface

Over the past thirty years there has been an increasing number of contributions to the topic of searching, both in theoretically as well as in applications-oriented journals. It is remarkable that very different kinds of problems are classified as search problems and that researchers from various fields often know little about results achieved in areas with which they are not familiar.

In this book we attempt to present the extensive literature in such a way that the reader can quickly understand the range of questions and obtain a survey of them which is comprehensive as possible.

Our goal was to treat the most important works in the field from the viewpoint of the current state of the science. However, we make no claim at all at completeness since the restraints of this book make such an attempt futile. For some works, which actually deserved to be presented in detail, we therefore settled for presenting the results. The interested researcher will then be in a position to find his own path through the literature. The book should be useful to the expert as a reference.

But our primary concern is to provide to every reader who is willing and able to engage in abstract, formal thought access to the basic ideas, methods and results in the field which have not yet appeared in book form but which deserve broader distribution due to their importance. Part 2 of the book is intended primarily for this broader group of readers. The only requirement here is a solid background in basic mathematics. All proofs are presented in great detail.

Knowledge of elementary probability theory is necessary to understand Part 4. Part 3 requires knowledge equivalent to a basic course in stochastics. In both these Parts the proofs are more concise.

All the individual Parts can be read and used independently for teaching purposes.

We wish to thank Mrs Waltraud Blenski for typing the manuscript.

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## Preface to the English Edition

The German edition of the book appeared in 1979. In 1982 the Russian edition was published by MIR. It includes also a supplement, *Information-theory Methods in Search Problems*, which was written by Maljutov and contains primarily those results obtained in the Soviet Union, to which we had no access. There, in particular, fascinating connections between multi-user information theory and the theory of screening design of experiments are emphasized. We think that some readers will appreciate that we included here the references from that supplement.

In recent years some striking improvements of earlier results have been obtained and, even more importantly, new connections of search to other areas and also new types of search problems have come into focus.

Under the heading 'Further reading' we mention articles and books which inform the researcher about new developments and results which seem to carry the seed for further discoveries. This selection certainly reflects the authors' research interests and judgements. However, we feel that an attempt to give some orientation to the reader, even at the risk of being proved wrong at a later time, is more valuable than an encyclopedic collection. It is conceivable that there is important work in progress which escaped our attention. We are grateful for any advice in this and other respects, which could be considered in another edition.

A systematic treatment of all the material referred to would require a major revision and extension of the book. This would not only cause further delay of this English edition, but also would not conform with the mostly elementary character of the book. Among conflicting demands we have kept as our highest priority the readability of the text for a large community. The time for another book will have come when most search problems can be put into the frame of a unified theory of search.

It is a pleasure to thank on this occasion Prof. M.V. Maljutov for his supplement of the Russian Edition. We are indebted to John Wiley & Sons Ltd for including the book in their series and to Dr. Spuhler from B.G. Teubner of Stuttgart as well as to Mrs C. Farmer from John Wiley & Sons Ltd for all their help and patience during the course of publication. We also thank Mrs M. Matz for her secretarial assistance.

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