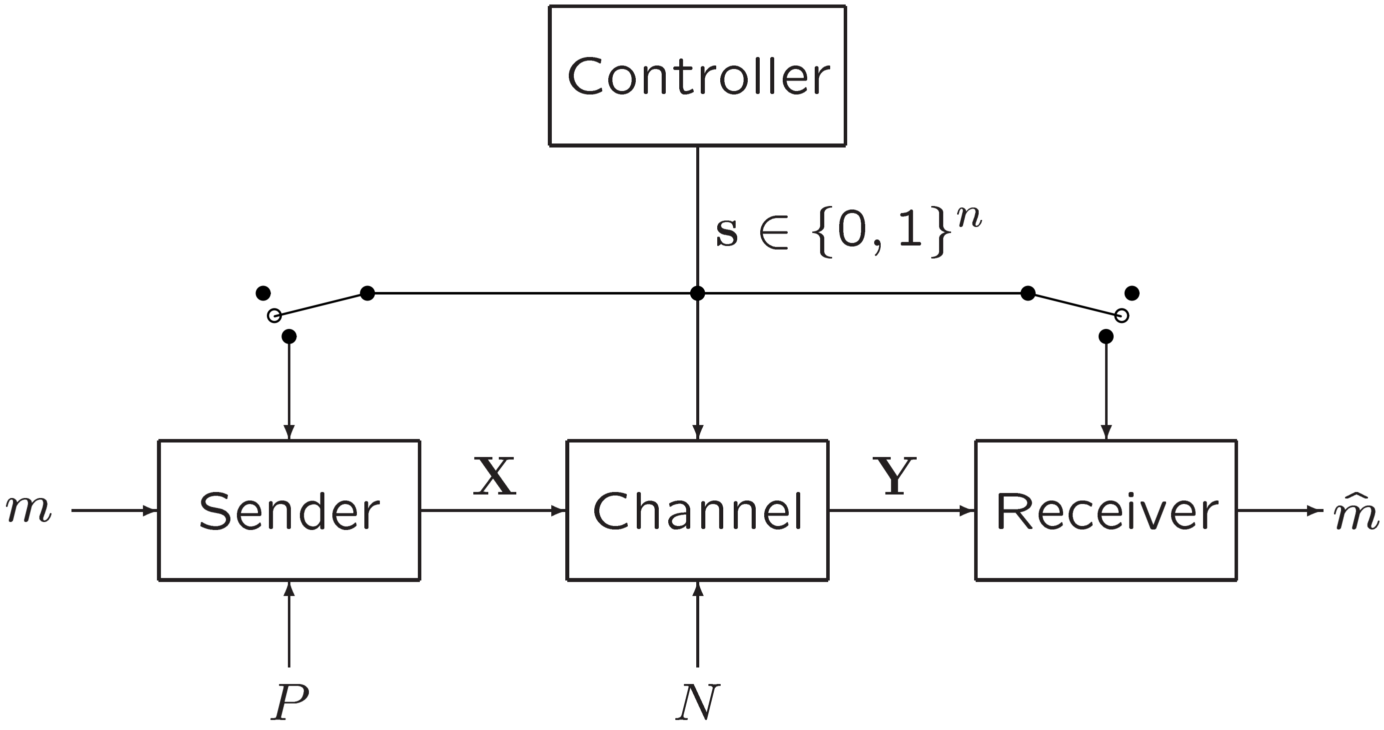
Communication over Time–Varying Channels under Power Constraints and Side Information of the Participants

Vladimir B. Balakirsky and A. J. Han Vinck
Institute for Experimental Mathematics
University of Essen
45326 Essen, Germany

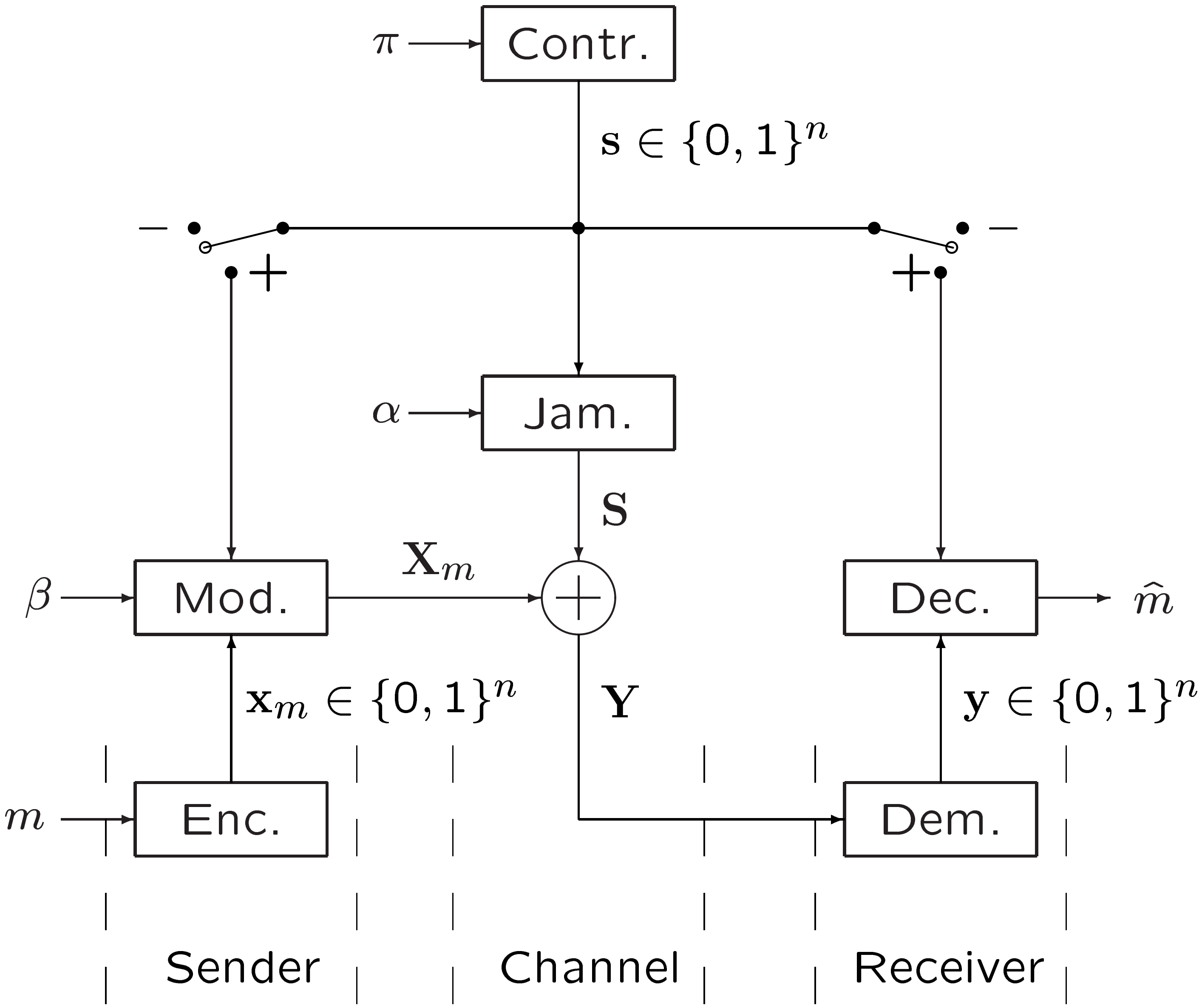
Motivation

- Like in xDSL, information about the channel conditions at the sender's or/and at the receiever's sides improves the performance.

- Power constraints are important for future communication systems, which use EMC.



Model for data transmission system with side information and power constraints



Formal model for data transmission

The received vector

$$\mathbf{Y} = \mathbf{X}_m + \mathbf{S}$$

is mapped by the Demodulator to the binary vector $\mathbf{y} = (y_1, \ldots, y_n)$, where

$$y_j = \begin{cases} 1, & \text{if } Y_j > 0 \\ 0, & \text{if } Y_j \le 0 \end{cases}$$

and $\mathbf{y}$ is the received vector for the Decoder.

The signal–to–noise ratio :

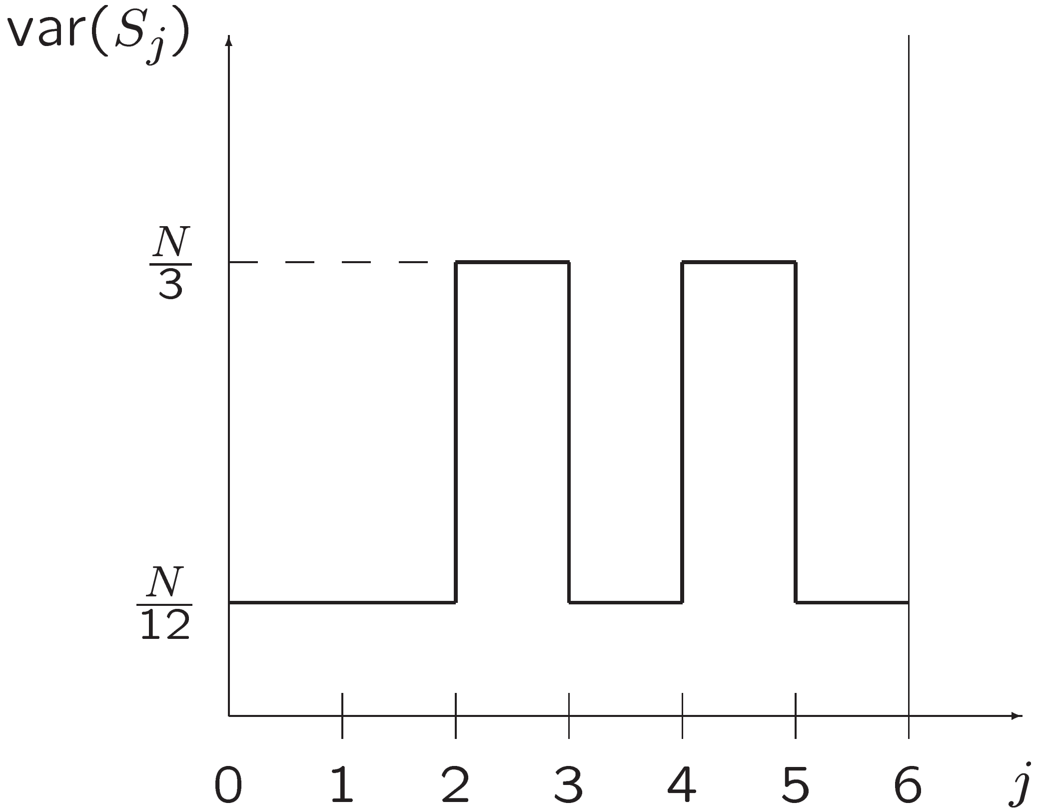
$$\rho \stackrel{\triangle}{=} P/N$$

$$W(\mathbf{y}|\mathbf{x},\mathbf{s}) \stackrel{\triangle}{=} \prod_{j=1}^{n} W_{x_j}(y_j|s_j)$$

for all $\mathbf{x}, \mathbf{y}, \mathbf{s} \in \{0,1\}^n$, where

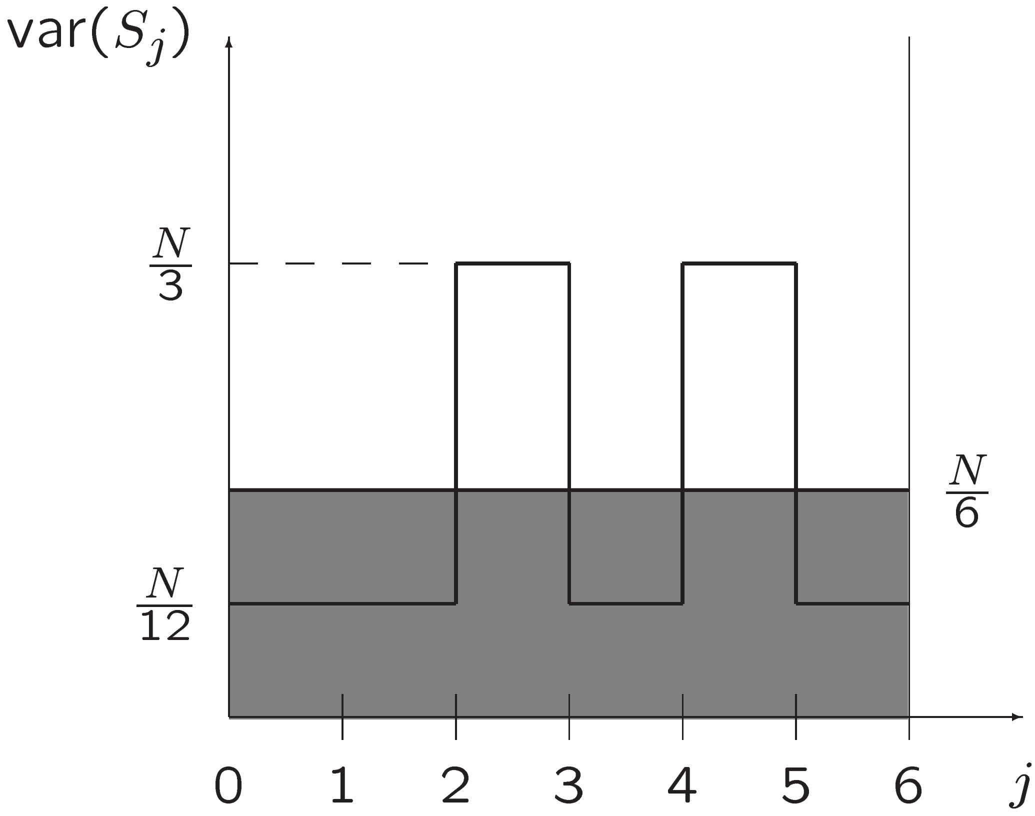
$$W_x(y|s) \stackrel{\triangle}{=} \begin{cases} 1 - p_{\mathrm{B}}, & \text{if } y = x \text{ and } s = 1 \\ p_{\mathrm{B}}, & \text{if } y \ne x \text{ and } s = 1 \\ 1 - p_{\mathrm{G}}, & \text{if } y = x \text{ and } s = 0 \\ p_{\mathrm{G}}, & \text{if } y \ne x \text{ and } s = 0 \end{cases}$$

Possible distribution of the variance of the noise, $n = 6$



$\pi = 2/6$ is the fraction of the number of "Bad" positions;
$\alpha = (2N/3)/N = 2/3$ is the fraction of the power of noise, which is put on "Bad" positions.

The uniform distribution of the power of the transmitted signal

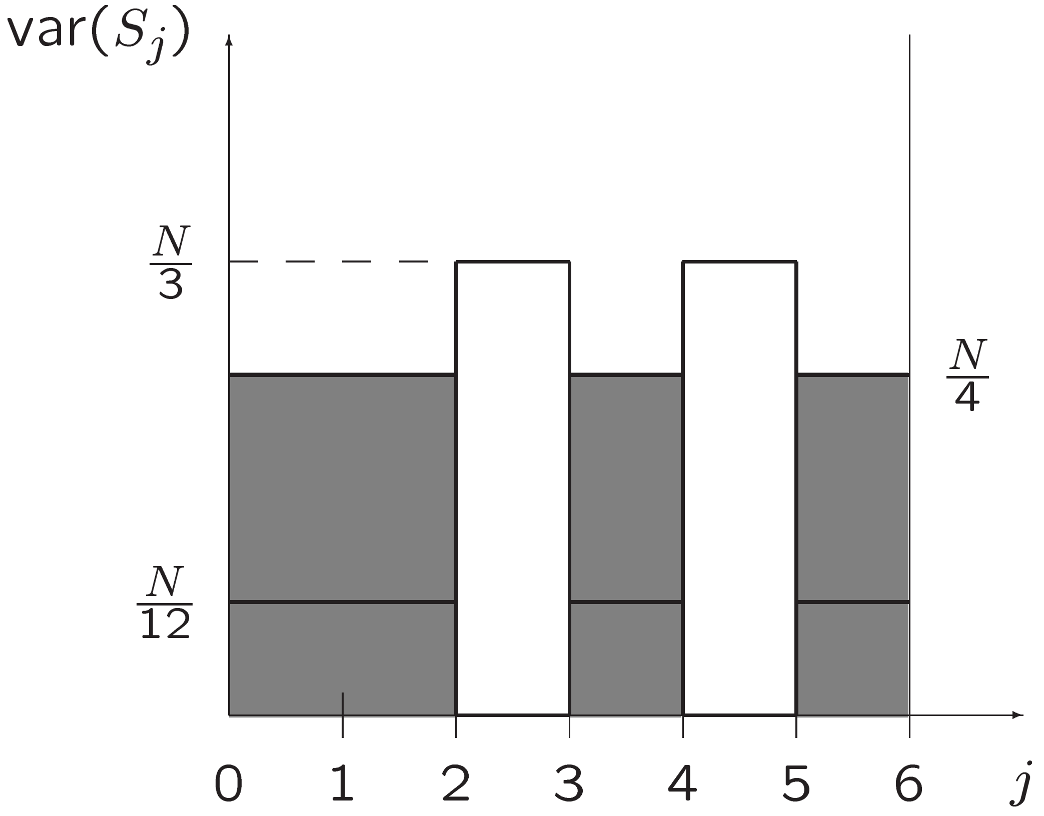


$$P = 6(N/6) = N$$

$$p_{\mathrm{B}} \;=\; \omega\left(\frac{N/6}{N/3}\right) = \omega(1/2)$$

$$p_{\mathrm{G}} \;=\; \omega\left(\frac{N/6}{N/12}\right) = \omega(2)$$

The distribution of the power of the transmitted signal, which is opposite to the distribution of noise

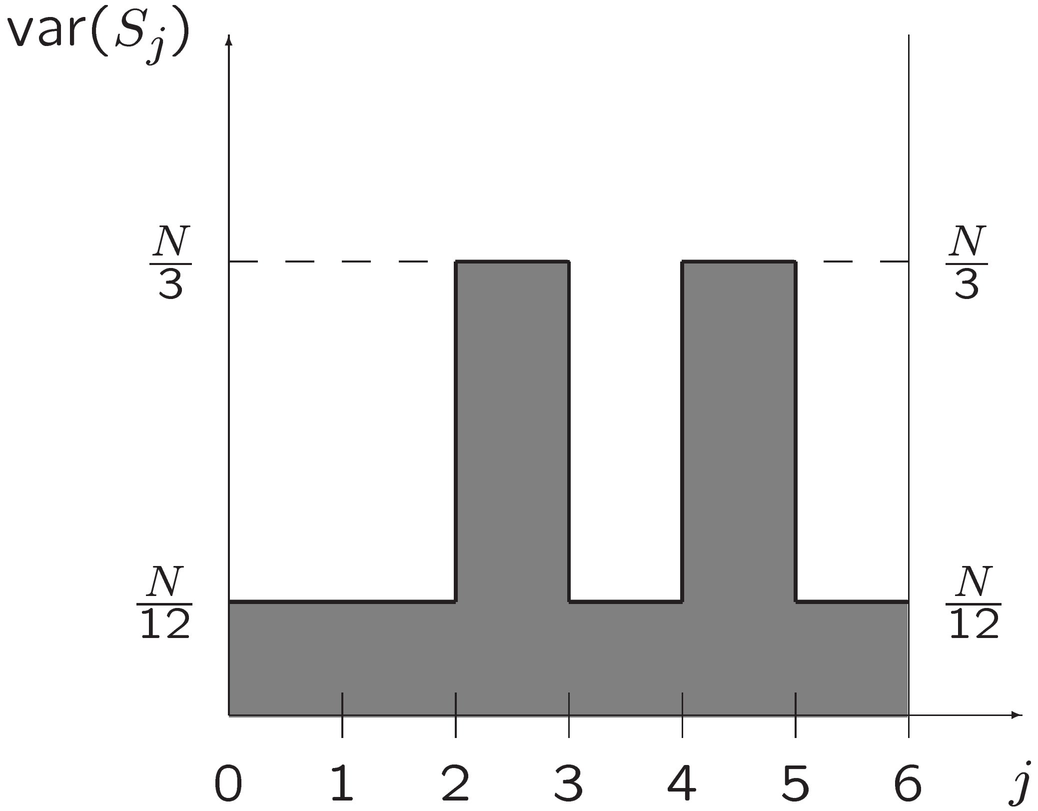


$$P = 4(N/4) = N$$

$$p_{\mathsf{B}} = \omega\left(\frac{0}{N/3}\right) = \omega(0)$$

$$p_{\mathsf{G}} = \omega\left(\frac{N/4}{N/12}\right) = \omega(3)$$

The distribution of the power of the transmitted signal, which is proportional to the distribution of noise



$$P = 2(N/3) + 4(N/12) = N$$

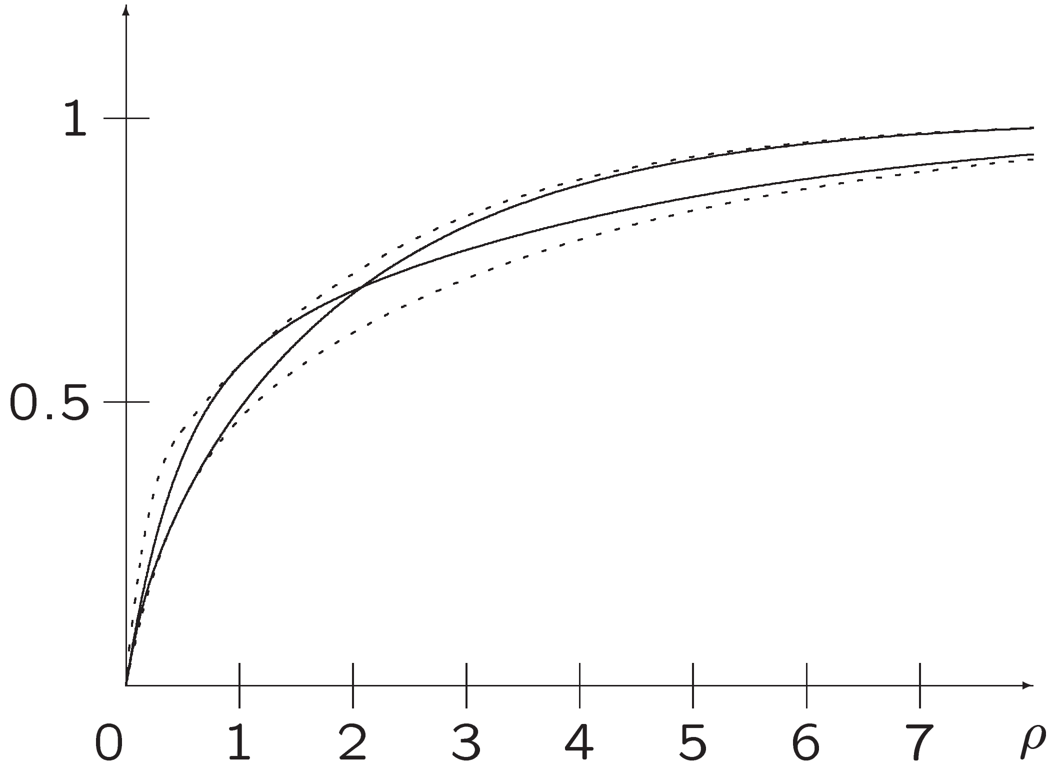
$$p_{\mathsf{B}} = \omega\left(\frac{N/3}{N/3}\right) = \omega(1)$$

$$p_{\mathsf{G}} = \omega\left(\frac{N/12}{N/12}\right) = \omega(1)$$

The order entries $\mathbf{y}$ of one coset of the standard array for the shortened Hamming $(6,3)$–code constructed for the state vector $\mathbf{s} = 001010$

$\mathbf{x}$	$p_{\mathrm{G}} q_{\mathrm{B}}^2 \geq q_{\mathrm{G}} p_{\mathrm{B}}^2$	$W(\mathbf{y} \mid 0 \ldots 0, \mathbf{s})$		$p_{\mathrm{G}} q_{\mathrm{B}}^2 < q_{\mathrm{G}} p_{\mathrm{B}}^2$
000000	00**0**0**0**1	$p_{\mathrm{G}} q_{\mathrm{G}}^3$	q_{B}^2	00**1**0**1**0
001011	00**1**0**1**0	q_{G}^4	p_{B}^2	00**0**0**0**1
010101	01**0**1**0**0	$p_{\mathrm{G}}^2 q_{\mathrm{G}}^2$	q_{B}^2	01**1**1**1**1
011110	01**1**1**1**1	$p_{\mathrm{G}}^3 q_{\mathrm{G}}$	p_{B}^2	01**0**1**0**0
100110	10**0**1**1**1	$p_{\mathrm{G}}^3 q_{\mathrm{G}}$	$p_{\mathrm{B}} q_{\mathrm{B}}$	10**1**1**0**0
101101	10**1**1**0**0	$p_{\mathrm{G}}^2 q_{\mathrm{G}}^2$	$p_{\mathrm{B}} q_{\mathrm{B}}$	10**0**1**1**1
110011	11**0**0**1**0	$p_{\mathrm{G}}^2 q_{\mathrm{G}}^2$	$p_{\mathrm{B}} q_{\mathrm{B}}$	11**1**0**0**1
111000	11**1**0**0**1	$p_{\mathrm{G}}^3 q_{\mathrm{G}}$	$p_{\mathrm{B}} q_{\mathrm{B}}$	11**0**0**1**0
		$q_{\mathrm{B}} = 1 - p_{\mathrm{B}}$ $q_{\mathrm{G}} = 1 - p_{\mathrm{G}}$		

Capacities of the communication system as functions of the signal–to–noise ratio ρ for $\pi = 0.5$ and $\alpha = 0.9$, where π is the fraction of the number of "Bad" positions and α is the fraction of the power of noise, which is put on "Bad" positions



$$\rho < 2.2 \implies C_{\pi,\alpha}^{(+,-)} < C_{\pi,\alpha}^{(-,+)}$$
$$\rho > 2.2 \implies C_{\pi,\alpha}^{(+,-)} > C_{\pi,\alpha}^{(-,+)}$$