A Class of Block Codes and Their Decoding for Transmission without Synchronization

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Abstract — **A class of block codes, based on properties of rooted, ordered, oriented trees, is examined for application to transmission without synchronization.**

We consider the following problem. Let M be a number of messages that should be transmitted over a noiseless binary channel, and suppose that there is no synchronization between the sender and receiver. Suppose that the sender uses a block code C of length n and that the decoder receives a binary sequence $\mathbf{y}=(y_0,...,y_{2n-2})$ containing the transmitted codeword $\mathbf{c}=(c_0,...,c_{n-1})$, i.e., $y_{k+t}=c_t, \quad t=0,...,n-1$ for some $k\in[0,...,n-1]$. Other positions of the vector $\mathbf{y}$ are filled at random. The decoder must uniquely find k and the transmitted message such that the decoding error probability is equal to zero. We propose a class of codes, based on properties of rooted, ordered, oriented trees [1], having a size asymptotically coinciding with 2^n, that may be decoded by an algorithm with linear complexity.

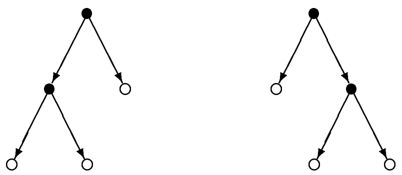
As well-known, there are

$$M=\frac{2}{n+1}\cdot\left(\begin{array}{c} n-1 \\ (n-1)/2 \end{array}\right)=\frac{1}{n}\cdot\left(\begin{array}{c} n \\ (n+1)/2 \end{array}\right) \qquad (1)$$

full, ordered, oriented, binary trees having n nodes; two trees with $n=5$ and their preorder codewords are given in Fig.1. Any tree may be defined by a preorder codeword of length n, composed of symbols 0 and 2. We construct a code C in such a way that it consists of the preorder codewords of all full, ordered, oriented, binary trees having n nodes, where each symbol 2 is replaced with 1; if $n=5$, then $C=(11000, 10100)$. Note that the set of all codewords may be obtained for any odd n using a simple computer algorithm [2]. Besides, there exist algorithms of direct generating of the codeword with a given number and vice versa [3].

The decoding algorithm, described below, at each time instant replaces the string 100 in $\mathbf{y}$ with 0. This operation may be realized using array $\mathbf{b}$, containing Boolean variables 0 and 1.

1. Set $b_t=1$ for all $t=0,...,2n-2$, and $b_{2n-1}=1$. Set $k=2n-4$.
2. Let $k_1>k$ and $k_2>k_1$ be indices such that $b_{k_1}=b_{k_2}=1$, and $b_t=0$ for all $t\in\{k+1,...,k_2-1\}\backslash k_1$. If $(y_k,y_{k_1},y_{k_2})\neq(1,0,0)$, then go to 5.
3. Set $b_{k_1}=b_{k_2}=0, \quad y_k=0$.
4. Let $k'>k$ be the minimal index such that $b_{k'}=1$. If $k'-k=n$, then go to 6.
5. Set $k=k-1$ and go to 2.
6. End : the transmitted codeword is written from position k to position $k+n-1$ in the received sequence.



$\mathbf{c}=(2,2,0,0,0)$ $\qquad$ $\mathbf{c}=(2,0,2,0,0)$

Figure 1: Full, ordered, oriented, binary trees with $n=5$ and their preorder codewords.

The current content of $\mathbf{b}$ and $\mathbf{y}$ in step 3 of the decoding procedure for the code C of length 5 is given below for a specific example.

$$\begin{array}{lcll} \mathbf{b} & = & (1,1,1,1,1,1,1,1,1,1), \quad \Rightarrow & (k,k_1,k_2,k')= \\ \mathbf{y} & = & (1,0,1,0,0,1,0,0,0), & =(5,6,7,8) \end{array}$$

$$\begin{array}{lcll} \mathbf{b} & = & (1,1,1,1,1,1,0,0,1,1), \quad \Rightarrow & (k,k_1,k_2,k')= \\ \mathbf{y} & = & (1,0,1,0,0,0,0,0,0), & =(2,3,4,5) \end{array}$$

$$\begin{array}{lcll} \mathbf{b} & = & (1,1,1,0,0,1,0,0,1,1), \quad \Rightarrow & (k,k_1,k_2,k')= \\ \mathbf{y} & = & (1,0,0,0,0,0,0,0,0), & =(0,1,2,5) \end{array}$$

$$k'-k=5-0=n.$$

Note that the decoder has to wait for the last bit of the received sequence to start the operations. If the delay is critical, then another version of the decoding algorithm may be used. Steps $1,3,4$ and 6 of this version coincide with the previous ones, but steps 2 and 5 are transformed into $2'$ and $5'$, respectively.

$2'$. Find any triple (k,k_1,k_2) such that $k<k_1<k_2$; $b_k=b_{k_1}=b_{k_2}=1$; $b_t=0$ for all $t\in\{k+1,...,k_2-1\}\backslash k_1$; $(y_k,y_{k_1},y_{k_2})=(1,0,0)$.

$5'$. Go to $2'$.

Theorem. Any code of length n with M codewords, where M is given in (1), consisting of preorder codewords of all full, ordered, oriented, binary trees with n nodes, provides a solution to the synchronization problem in conjunction with the decoding algorithms given above.

References

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