INEQUALITIES FOR LINEAR EXTENSIONS AND BALANCING PAIRS

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In a finite partially ordered set, $Prob(x > y)$ denotes the proportion of linear extensions in which element x appears above element y. In 1969, S. S. Kislitsyn conjectured that in every finite poset which is not a chain, there exists a pair (x, y) for which $1/3 \leq Prob(x > y) \leq 2/3$. In 1984, J. Kahn and M. Saks showed that there exists a pair (x, y) with $3/11 < Prob(x > y) < 8/11$,

We discuss inequalities for linear extensions of partially ordered sets, and give new proofs of some known inequalities. Special cases of the Cross Product Conjecture will be mentioned, in particular the case leading to an improvement over the Kahn-Saks result: Every poset which is not a chain contains a $(5 - \sqrt{5})/10$ balanced pair, i.e., a pair (x, y) for which $(5 - \sqrt{5})/10 \leq Prob(x > y) \leq (5 + \sqrt{5})/10$.