HANKEL MATRICES IN CODING THEORY AND COMBINATORICS

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We shall discuss Hankel matrices of the form

$$A_n^{(k)} = \begin{pmatrix} c_k & c_{k+1} & c_{k+2} & \cdots & c_{k+n-1} \\ c_{k+1} & c_{k+2} & c_{k+3} & \cdots & c_{k+n} \\ c_{k+2} & c_{k+3} & c_{k+4} & \cdots & c_{k+n+1} \\ \vdots & \vdots & \vdots & & \vdots \\ c_{k+n-1} & c_{k+n} & c_{k+n+1} & \cdots & c_{k+2n-2} \end{pmatrix}.$$

If the entries $c_m = \frac{1}{2m+1}\binom{2m+1}{m}$ are Catalan numbers then $\det(A_n^{(k)}) = \prod_{1\leq i\leq j\leq k-1} \frac{i+j+2n}{i+j}$ and for the binomial coefficients $c_m = \binom{2m+1}{m}$ it is $\det(A_n^{(k)}) = \prod_{1\leq i\leq j\leq k} \frac{i+j-1+2n}{i+j-1}$.

We shall also study Hankel determinants whose entries are defined as generalized Catalan numbers $c_m = \frac{1}{3m+1}\binom{3m+1}{m}$. In this case we could show that

$$\det(A_n^{(0)}) = \prod_{j=0}^{n-1} \frac{(3j+1)(6j)!(2j)!}{(4j+1)!(4j)!}, \qquad \det(A_n^{(1)}) = \prod_{j=1}^{n} \frac{\binom{6j-2}{2j}}{2\binom{4j-1}{2j}}.$$

These numbers are of special interest, since they occur in the enumeration of certain plane partitions and alternating sign matrices. Hankel matrices have important applications, for instance, in the theory of moments, and in Padé approximation. In Coding Theory Hankel matrices occur in the analysis of BCH - codes where the Berlekamp – Massey algorithm yields a fast decoding procedure.