Übungen zur Informationstheorie und Komplexität

Blatt 1

SS '98

1. Let $\mathcal{X}$ a finite set. A (nosequential, binary) search strategy $s = (t_1, ..., t_m)$ is a (finite) sequence of tests $t_i : \mathcal{X} \to \{0,1\}$.
   If it is 1-1 we call s (which is a map $s : \mathcal{X} \to \{0,1\}^m$) successful.
   Calculate
   $$m(\mathcal{X}) \triangleq \min\{m \in \mathbb{N} : \exists \text{ successful strategy } s = (t_1, ..., t_m)\} .$$

2. Define a (one tape) Turing Machine which solves the problem "sum of two nonnegative integers".
   (First of all the problem has to be stated in the form $\Pi \subset \{0,1\}^* \times \{0,1\}^*$.)

3. For a finite set $M \subset \mathbb{N}$ and some binary sequence $b^M = (b_i)_{i \in M}$ we define
   $$\overline{b^M} \triangleq \{(a_i)_{i\in\mathbb{N}} : a_i = b_i \, \forall \, i \in M\} \subseteq \{0,1\}^\infty \qquad \text{and} \qquad \mu(\overline{\mathrm{b^M}}) \triangleq 2^{-|\mathrm{M}|} .$$
   Let $\rho : \{0,1\}^\infty \to [0,1]$ the map with $(a_i)_{i\in\mathbb{N}} \mapsto \sum_{i\in\mathbb{N}} a_i 2^{-i}$.
   Prove the following assertions if possible:
   (a) ρ is 1-1.
   (b) ρ 'is' an isomorphism of the probability spaces $(\{0,1\}^\infty, \mu)$ and $([0,1], \lambda)$ with λ the Lebesgue measure on $[0,1]$ (i.e. $\lambda([a,b]) = b - a$ ...).

   (Even though we didn't speak about σ-algebras you should.)

Übung (bis auf weiteres): Di 12^{30}-14 , V2-216.