Übungen zur Informationstheorie und Komplexität

Blatt 2

SS '98

1. Let X a random variable. Prove:
   (a) For $\alpha > 0$, and if $X \geq 0$, then $\Pr\{X \geq \alpha\} \leq \dfrac{\mathbf{E}X}{\alpha}$ (*Markov* inequality).
   (b) Deduce that for $\epsilon > 0$: $\Pr\{|X - \mathbf{E}X| \geq \epsilon\} \leq \dfrac{\mathrm{Var}X}{\epsilon^2}$ (*Chebyshev* inequality).

2. Let $X_1, X_2, \ldots$ and X random variables on the probability space Ω. Prove that $\Pr\{\omega : \lim_{n\to\infty} X_n(\omega) = X(\omega)\} = 1$ (almost sure convergence of X_n) implies $\forall \epsilon > 0 \ \lim_{n\to\infty} \Pr\{|X_n - X| \geq \epsilon\} = 0$ (stochastic convergence of X_n).
   Give an example that the converse assertion is not true.

3. Recall that an infinite sequence $a \in \{0,1\}^{\mathbb{N}}$ is called *normal*, if for all $b \in \{0,1\}^*$ with length $|b|$: $\lim_{n\to\infty} \dfrac{< a^n | b >}{n} = 2^{-|b|}$.
   (a) Prove that the set of normal sequences has μ–measure 1.
   Hint: denote A_b the set of sequences which satisfy the normality condition for a specific b. Prove first that $\forall b \ \mu(A_b) = 1$.
   (b) The number 0.1234567891011121314... is called the (decimal) *Mahler number*. Prove that the fractional part of the binary Mahler number 0.1101110010111011111000... is normal, but not von Mises–Wald–Church random.

Übung (bis auf weiteres): Di 12^{30}-14 , V2-216.