Übungen zur Informationstheorie und Komplexität

Blatt 4

SS '98

1. Show that for every string x there exists a universal partial recursive function ϕ_x with $C_{\phi_x}(x) = 0$. Why does this not mean that Kolmogoroff complexity is a trivial concept?

2. Prove the following inequalities:

   (a) $C(x|x) \leq O(1)$.

   (b) $C(x,y) \leq C(y,x) + O(1)$.

   (c) $C(x,y) \leq C(x) + C(y|x) + 2l(x) + O(1) \leq C(x) + C(y) + 2l(x) + O(1)$.

   (d) *Continuity*: $|C(x+y) - C(x)| \leq 2l(y) + O(1)$.

3. Show that exercise 1.(b) is essentially best possible, in that we cannot get rid of the $l(x)$ term for nothing (resulting in a subadditivity property $C(x,y) \leq C(x) + C(y) + O(1)$): for every n there exist strings x, y of length at most n with $C(x) + C(y) + l(n) \leq C(x,y) + O(1)$.
   (Hint: Consider all pairs (x,y) with $n = l(x) + l(y)$ and prove that there is one with $C(x,y) \geq n + \log n$. Then combine with a trivial upper bound on $C(x) + C(y)$).

Übung (bis auf weiteres): Di 12^{30}-14 , V2-216.