Übungen zur Informationstheorie und Komplexität

Blatt 6

SS '98

1. Recall from the lecture that $v(n)$ is the maximal number k such that that in every tournament on n nodes there exists a transitive subtournament on k nodes. Prove that $v(n) \geq 1 + \lfloor \log n \rfloor$.
   Hint: use induction on n by proving that $v(n) \geq 1 + v\left(\left\lceil \frac{n-1}{2} \right\rceil\right)$.

2. Prove the following upper bound on the Ramsey number $r(k,l)$ (this is the least n such that every $\{\text{red}, \text{blue}\}$–coloring of the edges of the complete graph K_n on n vertices contains a red K_k or a blue K_l. In the lecture only $r(k,k)$ was defined):
$$r(k,l) \leq \binom{k+l-2}{k-1}$$
   (Compare this with Erdős's lower bound presented in the lecture! See also previous exercises).
   Hint: first consider the cases $k = 1$, $l = 1$, then use induction on $k + l$ by proving that $r(k,l) \leq r(k-1,l) + r(k,l-1)$.

3. Let X, Y random variables, and f a function defined on the range of Y, so that $Z = f(Y)$ is also a random variable. Prove that X, Y, Z is a Markov chain
$$X \multimap Y \multimap Z$$
   (i.e. $\Pr\{Z = z | Y = y, X = x\} = \Pr\{Z = z | Y = y\}$).

Übung: Di 12^{30}-14 , V2-216.
Nächste Woche (2.6.) Vorstellung von Diplomarbeitsthemen!
Abgabe deshalb bis 8.6.98.