## Übungen zur Informationstheorie und Komplexität

**Blatt 11**

**WS 1999/2000**

1. Denoting the maximum cardinality of an error–forgiving WUM of block length $n$ by $M(n)$, prove that $M(1) = 2$, $M(2) = 3$, $M(3) = 5$, and $M(4) = 7$.

2. Compute the rate of the WUMs constructed by use of Baranyai's theorem with parameter $k$, and find the $k$ for which the rate is largest.

3. (**Color carrying lemma**) Let $\Gamma = (\mathcal{V}, \mathcal{E})$ be a hypergraph such that for all $E \in \mathcal{E}$ $|E| \geq k$. Prove that there exists a coloring of the vertices of $\Gamma$ with

$$c = \left\lfloor \frac{k}{\ln(k \cdot |\mathcal{E}|)} \right\rfloor$$

   many colors such that in every edge all $c$ colors occur.
   (Hint: use the probabilistic method, coloring each vertex independently and uniformly).

4. Use the color carrying lemma to prove the direct parts of Borden's theorems on optimal WUM code rates:

   (a) For error–forgiving WUM codes consider the hypergraph $H_n$ with vertex set $\mathcal{V} = \{0,1\}^n$ and edge set $\mathcal{E} = \{K(x) : x \in \mathcal{V}\}$.

   (b) For general WUM codes restrict $H_n$ to the vertex set $\binom{[n]}{\alpha n} \cup \binom{[n]}{n-\alpha n}$, and optimize with respect to $\alpha$.

Übung: Mittwoch, 16ct, in U5–135.