Übungen zur Informationstheorie und Komplexität

Blatt 12

WS 1999/2000

1. Recall for a WEM code $\mathcal{C} = \{C_i : i = 1, \ldots, M\}$ on $\mathcal{X}$ and cost function $\varphi : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ the definition of the maximum transition cost

$$D_{\max}(\mathcal{C}) = \max_{1 \le i,j \le M} \max_{x^n \in C_i} \min_{y^n \in C_j} \varphi_n(x^n, y^n),$$

and that $\mathcal{C}$ is called a (n, M, D) code if $D_{\max}(\mathcal{C}) \le D$.

Define the optimum rates

$$R_n(d) = \max\{\frac{1}{n} \log M : \exists \text{ a } (n, M, nd) \text{ WEM code}\},$$

$$R(d) = \lim_{n \to \infty} R_n(d).$$

Prove that

   (a) $R_{n+m}(d) \ge \frac{n}{n+m} R_n(d) + \frac{m}{n+m} R_m(d)$ (compare with the behaviour of the analogous quantities in channel coding and rate distortion theory).

   (b) $R(d) = \sup\{\frac{1}{n} \log M : \exists \text{ a } (n, M, nd) \text{ WEM code}\}$.

2. Use the result of Ahlswede–Zhang to show for Hamming WEM codes that $R(d) = h(d)$, if $d \le 1/2$.

3. Prove that for a linear code C in $V = GF(2)^n$ the decoding rule

   The error pattern of a received word x^n is the minimal weight representative of the coset $x^n + C \in V/C$.

   is identical to the maximum likelihood decoding rule.

Übung: Mittwoch, 16ct, in U5–135.