Übungen zur Informationstheorie und Komplexität

Blatt 13

WS 1999/2000

1. For a finite dimensional Hilbert space $\mathcal{H}$ and linear operators $A, B \in \mathcal{L}(\mathcal{H})$ prove that $\mathrm{Tr}A^* = \overline{\mathrm{Tr}A}$ and $\mathrm{Tr}AB = \mathrm{Tr}BA$.

2. Recall the Bloch representation for states:

$$\rho[\mathbf{x}] = \frac{1}{2} \begin{pmatrix} 1 - x_1 & x_2 + ix_3 \\ x_2 - ix_3 & 1 + x_1 \end{pmatrix}, \text{ with } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

   which is a state iff $\mathbf{x}^2 \leq 1$.

   Show that for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$

$$\mathrm{Tr}\left(\rho[\mathbf{x}]\rho[\mathbf{y}]\right) = \frac{1}{2}(1 + \mathbf{x}\mathbf{y}),$$

   and use this formula to solve exercise 9.2.b.

3. Show that the probability postulate for POVMs entails the probability postulate for PVMs.

4. Show that the probability estimate for PVMs implies the expectation value postulate for an observable, using its spectral measure.

Übung: Mittwoch, 16ct, in U5–135.