Übungen zur Informationstheorie und Komplexität

Blatt 8

WS 1999/2000

1. A *rooted* tree is a tree with a distinguished vertex, called *root*. Two rooted trees are called *equivalent* if there is an isomorphism of them (as graphs) which matches their roots.

   Prove that the number of inequivalent rooted trees with n edges equals the *Catalan* number
$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

2. Consider a collection of rooted trees representing every equivalence class exactly once. Prove that exactly half of the total number of vertices are leaves (i.e. vertices of degree one).

3. For a permutation π of $\{1, 2, \ldots, n\}$ define
$$s(\pi) = \sum_{i=1}^{n} (\pi_i - \pi_{i+1})^2,$$
   with $\pi_{n+1} = \pi_1$.

   Determine the extreme values of s.

Übung: Mittwoch, 16ct, in U5–135.