Übungen zur Informationstheorie und Komplexität

Blatt 9

WS 1999/2000

1. Recall the generalized Harper's theorem:

   Let $r_1, \ldots, r_N$ be a nonincreasing sequence of nonnegative integers, and order the elements of $\{0,1\}^n$ first by the number of zeros, second lexicographically: $v_1 \succ v_2 \succ \cdots \succ v_{2^n}$. Then for any distinct $x_1^n, \ldots, x_N^n \in \{0,1\}^n$

   $$\left| \bigcup_{i=1}^{N} S_{r_i}(x_i^n) \right| \geq \left| \bigcup_{i=1}^{N} S_{r_i}(v_i) \right| .$$

   (a) Show by example that equality can occur for a family which does not consist of $v_1, \ldots, v_N$.

   (b) Show that for every N there exists N' such that

   $$\bigcup_{i=1}^{N} S_1(v_i) = \{v_1, \ldots, v_{N'}\}.$$

2. Consider $\mathbb{R}^2$ and $\mathbb{C}^2$ as real, respectively complex Hilbert space. Of course in both the maximum number of mutually orthogonal rays is 2. Relax the orthogonality condition to being at an angle of at least $\pi/4$:

   (a) Prove that in $\mathbb{R}^2$ the maximum number of unit vectors v_i such that $|\langle v_i, v_j \rangle| \leq 1/\sqrt{2}$ for $i \neq j$ is 4.

   (b) Construct six unit vectors $u_1, \ldots, u_6$ in $\mathbb{C}^2$ such that $|\langle v_i, v_j \rangle| \leq 1/\sqrt{2}$ for $i \neq j$. Can you find seven?

3. The new year's problem: What is easily caught, but never thrown?

Übung: Mittwoch, 16ct, in U5–135.