# Asymptotic bounds for the rate of coloring superimposed codes

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We introduce $q$-ary $(r_0, r_1, \ldots, r_{q-1})$ superimposed codes and give upper and lower asymptotic bounds for the rate of these codes.

**Definition 1.** *A $q$-ary matrix $C = \|c_{ij}\|$ of size $N \times T$ is a $(r_0, r_1, \ldots, r_{q-1})$ superimposed code if for any subsets $R_0, R_1 \ldots R_{q-1} \subset [T]$ with $|R_s| = r_s$ there exists a coordinate $i \in [N]$ such that $c_{ij} = s$ for all $j \in R_s$, where $s = 0, 1, \ldots, q-1$.*

Denote by $N(T, r_0, r_1, \ldots, r_{q-1})$ the minimum possible length of a $(r_0, r_1, \ldots, r_{q-1})$ superimposed code of a given cardinality $T$. The rate of a code of length $N$ and cardinality $T$ is, as usual, $R = (\log T)/N$. We are interested in the asymptotic behavior of the rate

$$R(r_0, r_1, \ldots, r_{q-1}) = \limsup_{T \to \infty} \frac{\log_q T}{N(T, r_0, r_1, \ldots, r_{q-1})}.$$

**Theorem 1.** *For coloring superimposed codes we have*

$$R(r_0, r_1, \ldots, r_{q-1}) \geq 1/(S-1) \log_q \frac{S^S}{S^S - r_0^{r_0} r_1^{r_1} \ldots r_{q-1}^{r_{q-1}}},$$

*where $S = (r_0 + r_1 + \ldots + r_{q-1})$.*

**Theorem 2.** *We have*

$$R(r_0, r_1, \ldots, r_{q-1}) \leq \frac{R(r_0 - x_0, r_1 - x_1, \ldots, r_{q-1} - x_{q-1})}{R(r_0 - x_0, \ldots, r_{q-1} - x_{q-1})/(1 - log_q(q-1)) + X^X/(x_0^{x_0} \ldots x_{q-1}^{x_{q-1}})}.$$

where $X = x_0 + x_1 + \ldots + x_{q-1}$.

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