Generic erasure correcting sets

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A generic (r, m)-erasure correcting set generates for each binary linear code of codimension r a collection of parity check equations that enables iterative decoding of all potentially correctable erasure patterns of size at most m. As we have shown earlier, such a set essentially is just a parity check collection with this property for the Hamming code of codimension r. We prove non-constructively that for fixed m the minimum size $F(r, m)$ of a generic (r, m)-erasure correcting set is linear in r. Moreover, we show constructively that $F(r, 3) \leq 3(r-1)^{\log_2 3} + 1$, which is a major improvement on a previous construction showing that $F(r, 3) \leq 1 + \frac{1}{2}r(r-1)$. In the course of this work we encountered the following problem that may be of independent interest: what is the smallest size of a collection $C \subset F_2^n$ such that, given any set of s independent vectors in F_2^n , there is a vector $c \in C$ that has inner product 1 with all of these vectors? We show non-constructively that, for fixed s, this number is linear in n.