

In this seminar we want to study (aspects of) the proof of the Tate conjecture for K3 surfaces by Madapusi Pera [MP15]. We will follow the exposition of Benoist [Be14], adding details as necessary.<sup>1</sup> The following list of talks is a suggestion. We may decide to split talks or add talks for more details.

- 1) Complex K3 surfaces: Basic properties and the Torelli Theorem. [Be14, 0.1] 0.1 and [Hu15]; details to be discussed. Note that [Be14, Definition 0.1] contains [Hu15, Proposition 2.4].
- 2) The Complex Kuga-Satake Construction. [Be14, 1.1] and [Hu15, 4.1-4.2, possibly 4.3-4.4].
- 3) K3 surfaces in positive characteristic. [Be14, 0.2-0.6]; one can also consult [Li16] and further literature as necessary.
- 4) Kuga-Satake Construction and period maps; Rationality. [Be14, 1.1-1.4] and [MP15, Prop. 5.3, Cor. 5.4, Prop. 5.6]. Construct the complex period map and Kuga-Satake map as a morphism of Shimura varieties; show that these descend to  $\mathbb{Q}$  using absolute Hodge cycles [DMOS].
- 5) Kuga-Satake Construction: Integral models and Torelli Theorem. [Be14, 1.5-1.6] and [MP15, Prop 5.7, Thm 5.8, Cor 5.15]. Among other things one uses the notion of healthy regular rings from [VZ10]. Note that [Be14, Th. 1.14] corresponds to [MP15, Cor. 5.15] because  $\mathfrak{M}_{2d}$  of [Be14] is  $M_{2d}^\circ$  of [MP15] (stack of polarised K3 surfaces).
- 6) Proof of the Tate conjecture. [Be14, 2.1-2.2, 2.4-2.5]

## Literatur

- [Be14] O. Benoist, Construction de courbes sur les surfaces K3, Seminaire Bourbaki 1081, 2014, <https://www.math.ens.psl.eu/~benoist/articles/K3.pdf>
- [DMOS] Deligne, Milne, Ogus, Shih, Hodge Cycles, Motives, and Shimura Varieties, 1982, <https://link.springer.com/content/pdf/10.1007/978-3-540-38955-2.pdf>

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<sup>1</sup>Note that the section numbering in the preprint version of [MP15] differs from the published version, and [Be14] refers to the preprint version. For example when [Be14] refers to [MP13, Proposition 4.6] this is [MP15, Proposition 5.6].

- [Hu15] D. Huybrechts, Lectures on K3 surfaces, 2015, <https://www.math.uni-bonn.de/people/huybrech/K3Global.pdf>
- [Li16] C. Liedtke, Lectures on Supersingular K3 Surfaces and the Crystalline Torelli Theorem, 2016, <https://link.springer.com/content/pdf/10.1007/978-3-319-29959-4.pdf>
- [MP15] K. Madapusi Pera, The Tate conjecture for K3 surfaces in odd characteristic, *Invent. Math.*, 2015, <https://link.springer.com/content/pdf/10.1007/s00222-014-0557-5.pdf>
- [VZ10] A. Vasiu, T. Zink, Purity results for  $p$ -divisible groups and abelian schemes over regular bases of mixed characteristic, *Doc. Math.* 15, 571–599 (2010), <https://www.math.uni-bielefeld.de/documenta/vol-15/17.pdf>